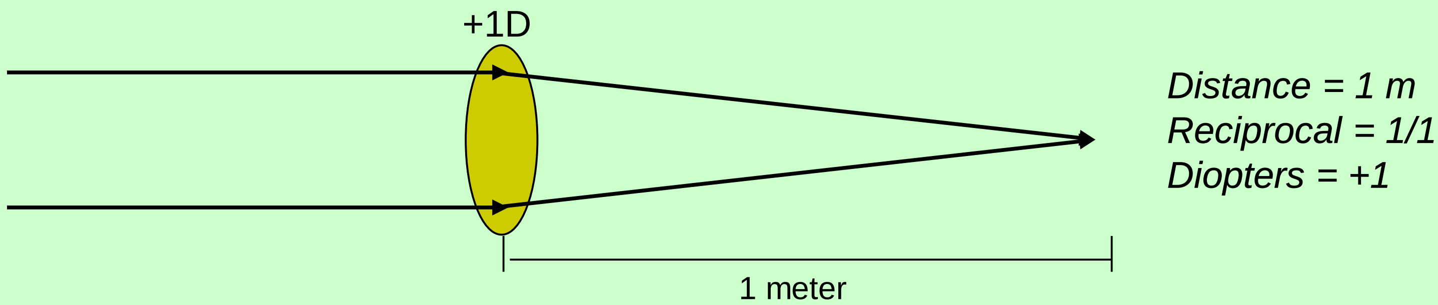
Vergence: The Vergence Formula

Basic Optics, Chapter 3

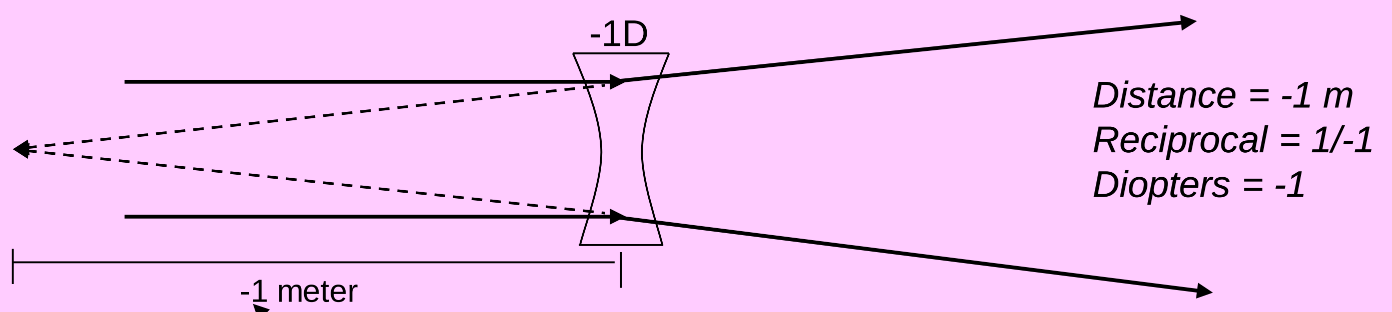
Vergence: The Vergence Formula

A **+1D** lens will focus parallel rays 1m to the right of the lens



We have seen how the dioptric power of a lens affects incoming parallel rays.

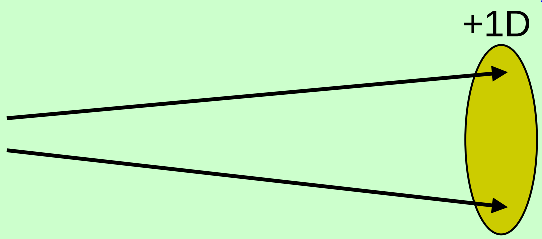
A **-1D** lens will ‘focus’ parallel rays1m to the left of the lens



(Remember, distances to the left of the lens are considered ‘minus’)

Vergence: The Vergence Formula

A **+1D** lens will focus non parallel rays **?**m to the **?** of the lens

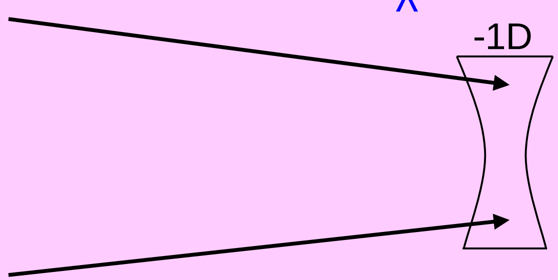


?

Diopters = +1
Reciprocal = ?
Distance = ?

We have seen how the dioptric power of a lens affects incoming parallel rays. *But what if the rays are not parallel?*

A **-1D** lens will 'focus' non parallel rays **?**m to the **?** of the lens



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Diopters = -1
Reciprocal = ?
Distance = ?

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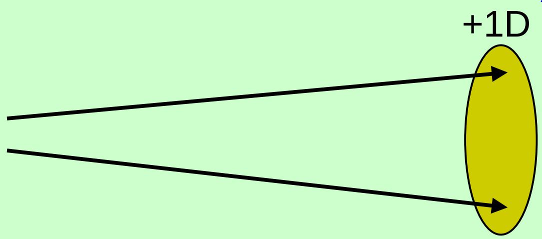

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Diopters = +1
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We have seen how the dioptric power of a lens affects incoming parallel rays. *But what if the rays are not parallel?* We need a more generalized concept concerning the relationships among incoming/outgoing rays, and lenses.

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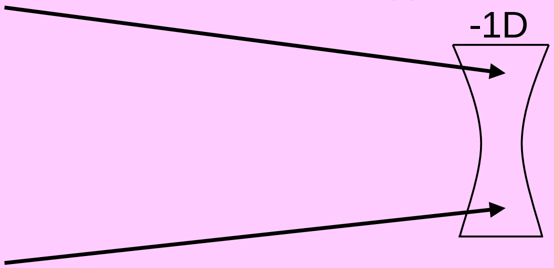

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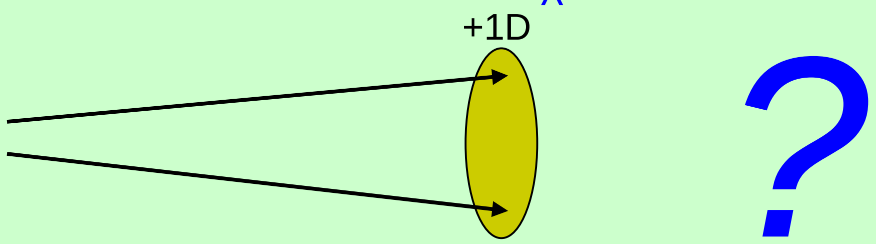

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We need a **more generalized concept** concerning the relationships among incoming/outgoing rays, and lenses.

This is provided by ***The Vergence Formula***

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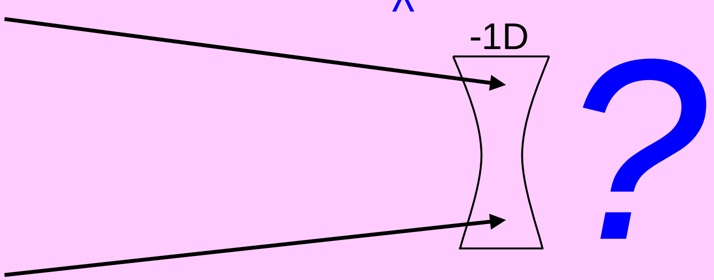

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Vergence: The Vergence Formula

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- *The Vergence Formula*
  - Crucial concept in optics
  - Describes the vergence relations among rays before, during and after encountering a refractive surface (e.g., lens)
    - *Head's up:* We will also use the Vergence Formula in describing the relations among rays interacting with *reflecting* surfaces, i.e., mirrors

Vergence: The Vergence Formula

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$$U + P = V$$

Vergence: The Vergence Formula

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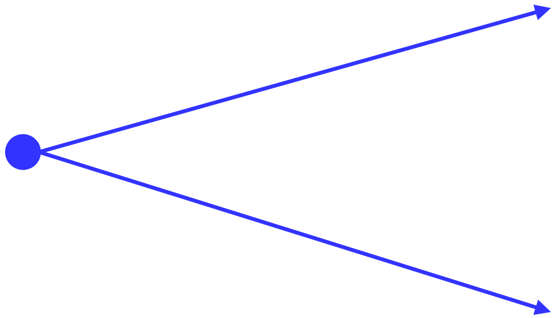
Vergence of
incoming light
(in diopters)

$$\mathbf{U} + P = V$$



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Vergence of incoming light (in diopters)

Vergence **contributed by lens** (in diopters)

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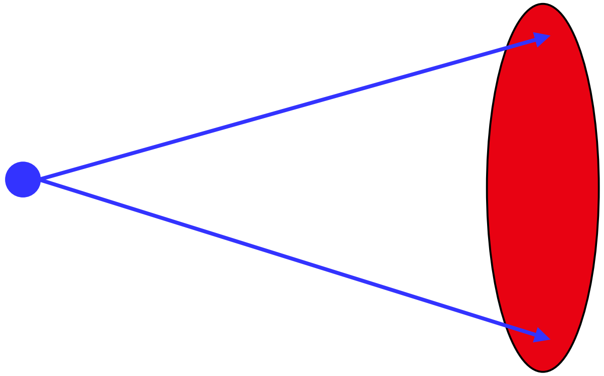

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Vergence of **light leaving lens** (in diopters)

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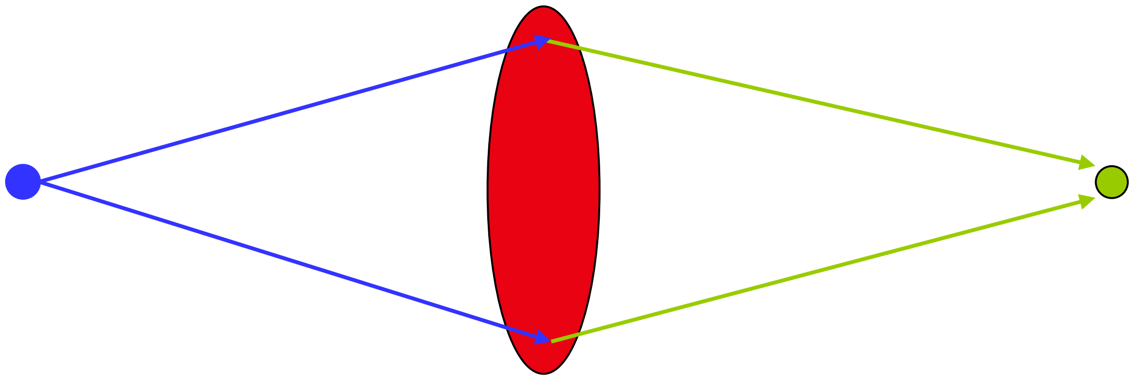

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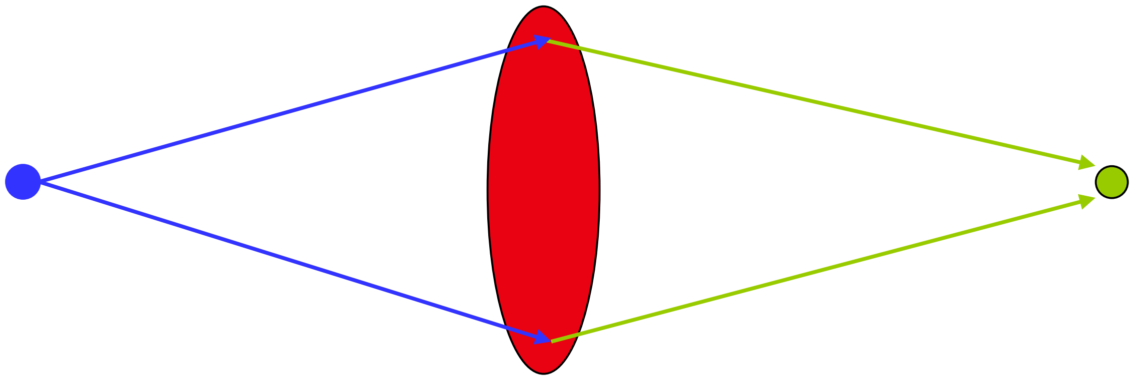

tl;dr The vergence of light leaving a lens is the sum of the vergence of the light entering the lens and the vergence contributed by the lens itself

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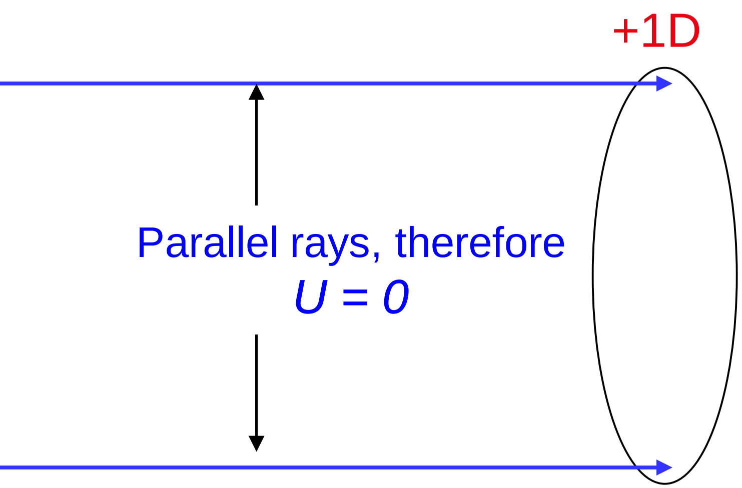

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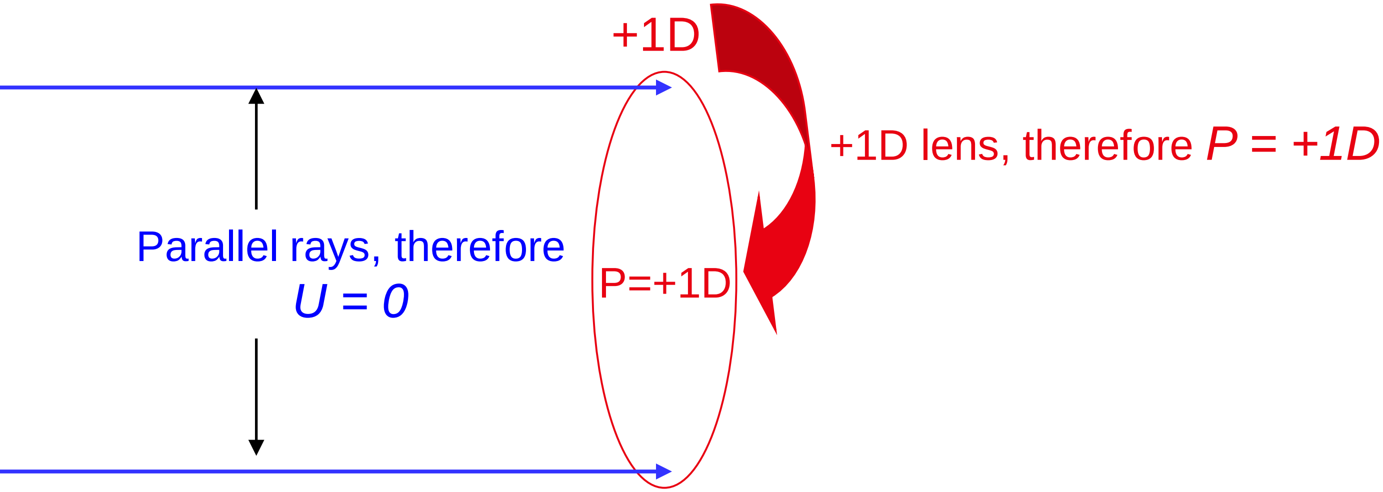

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Plugging these values into the Vergence Formula:

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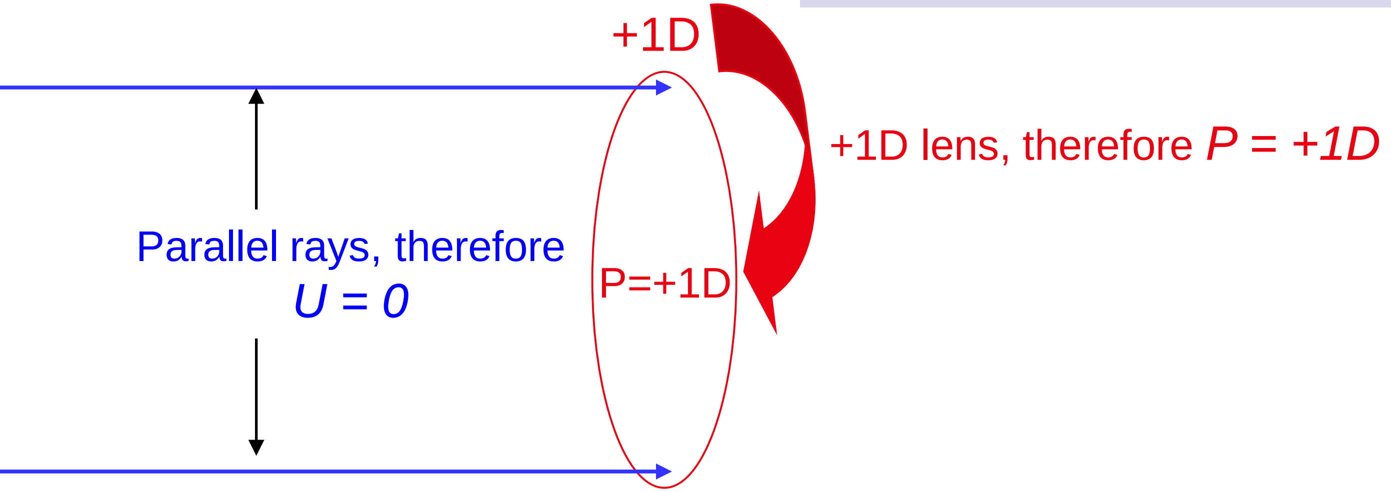

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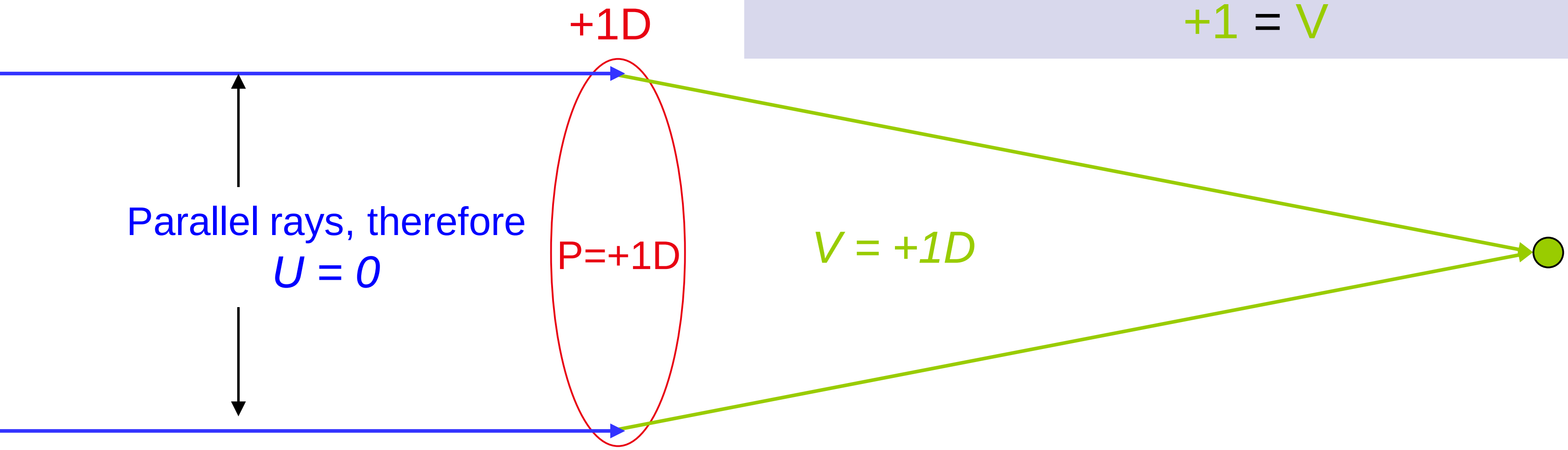

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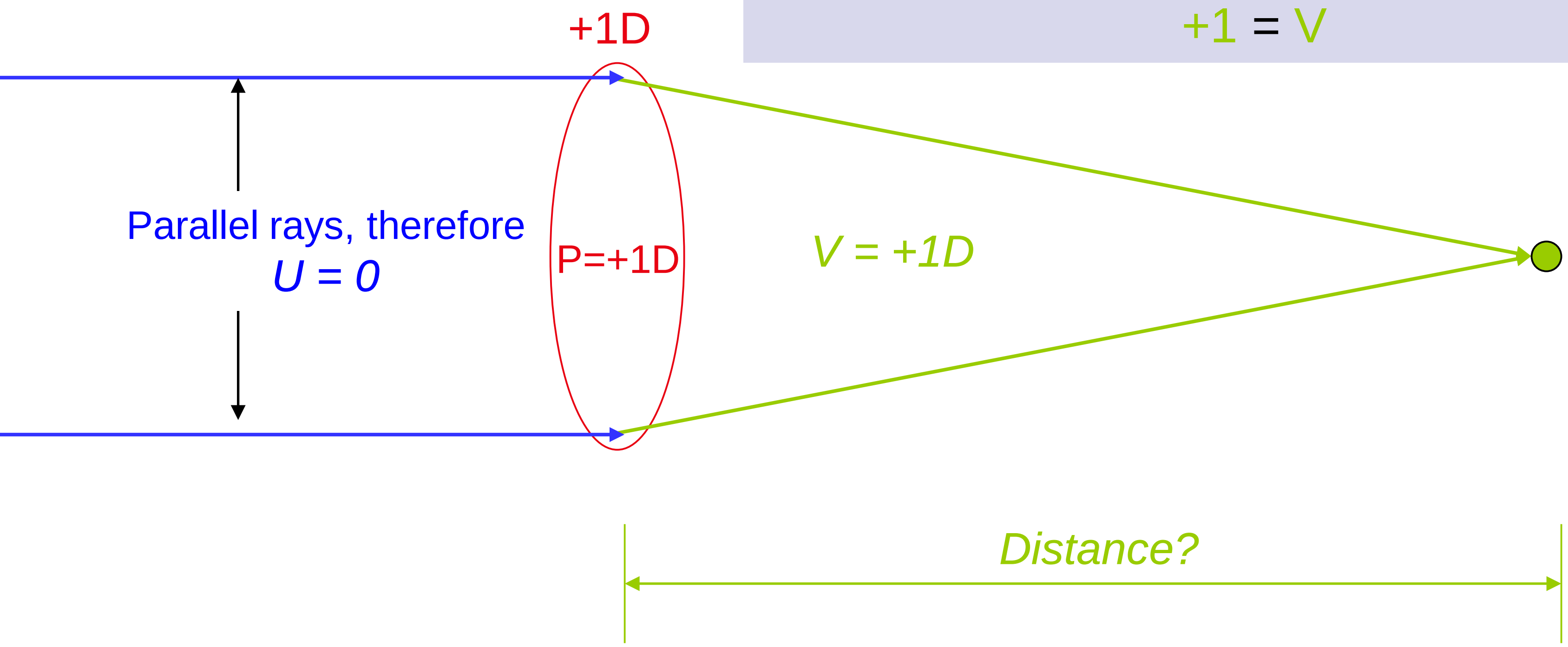

Distance equals the reciprocal of the outgoing vergence, ie, **1/V**

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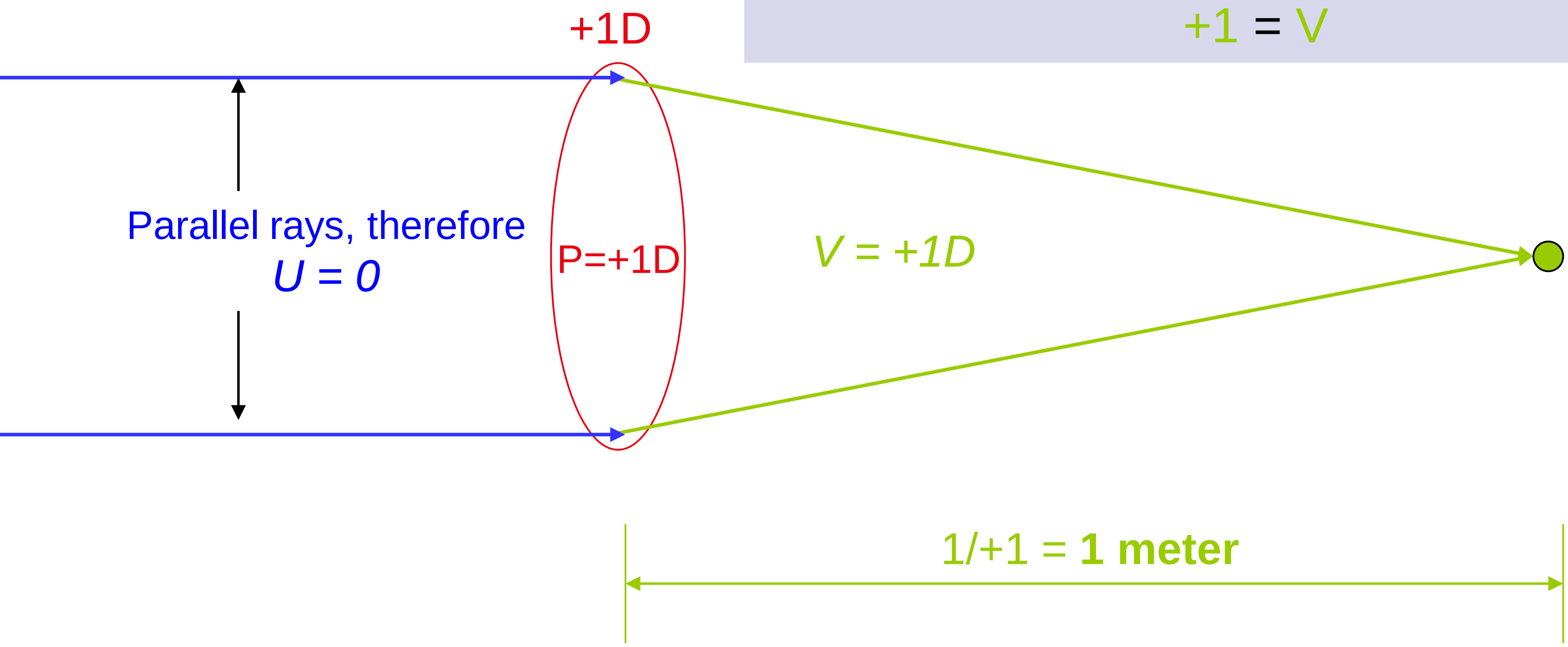

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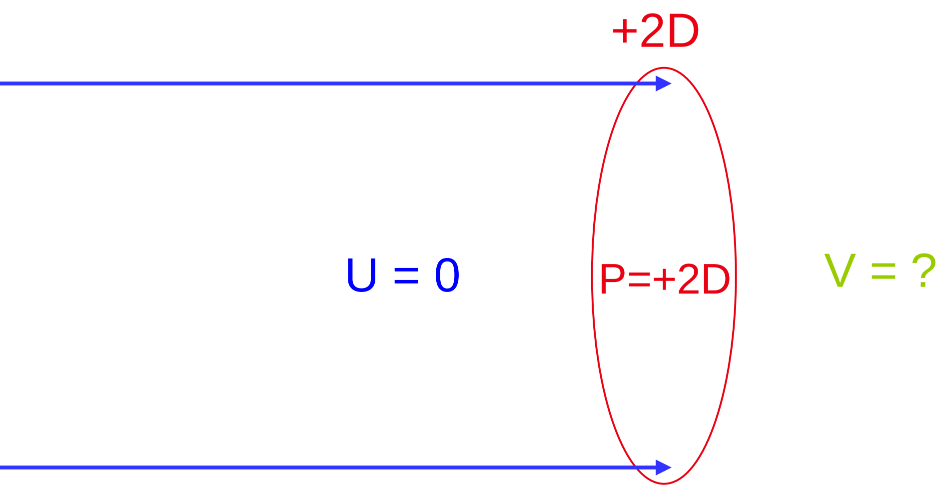

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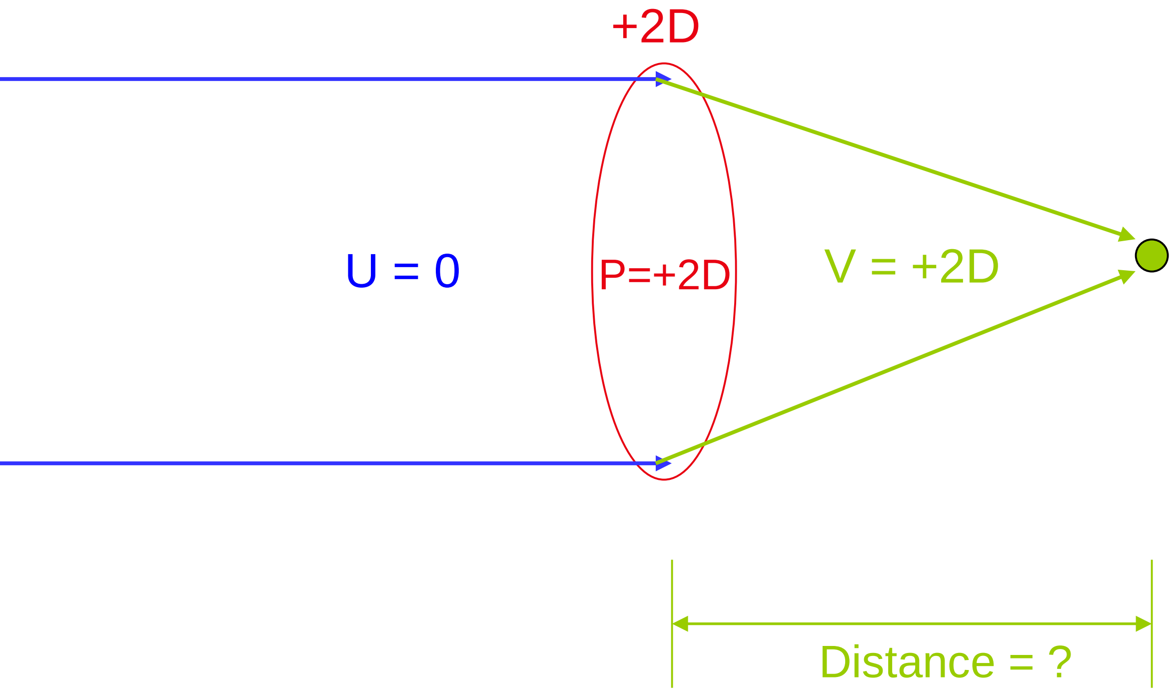

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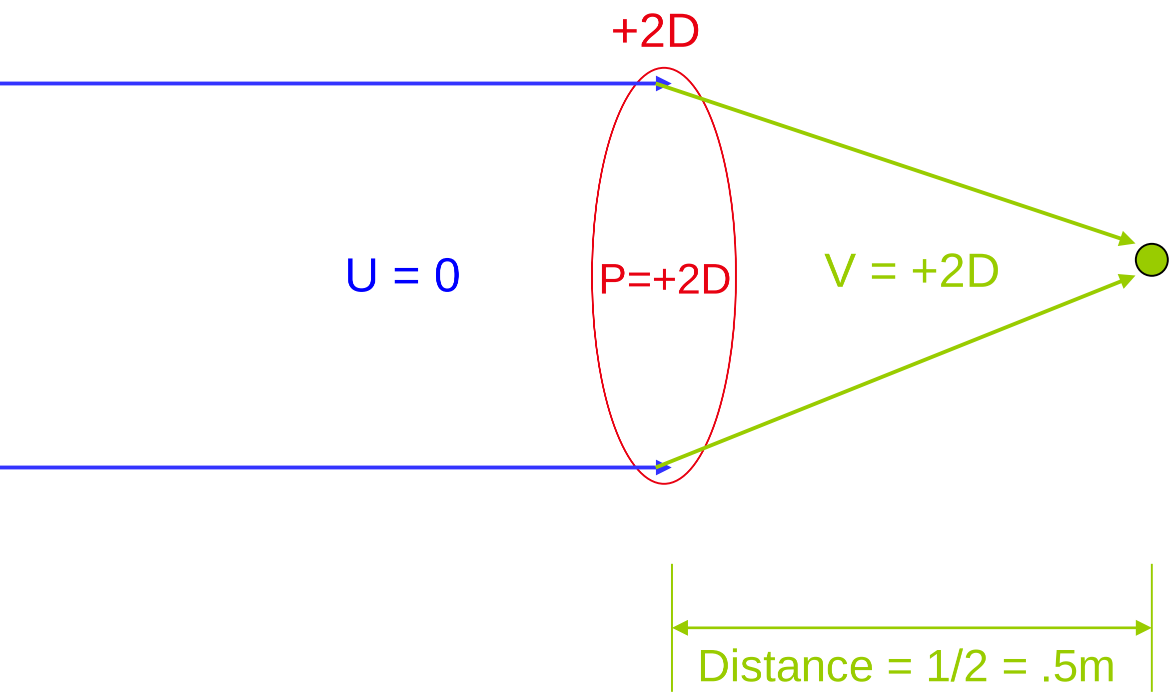

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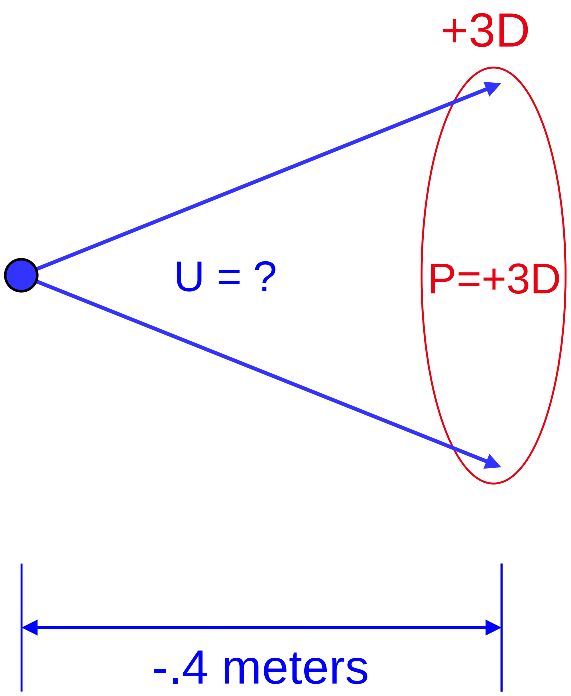

To determine the vergence ***U*** of the incoming light, take the reciprocal of the distance from its source or focal point:

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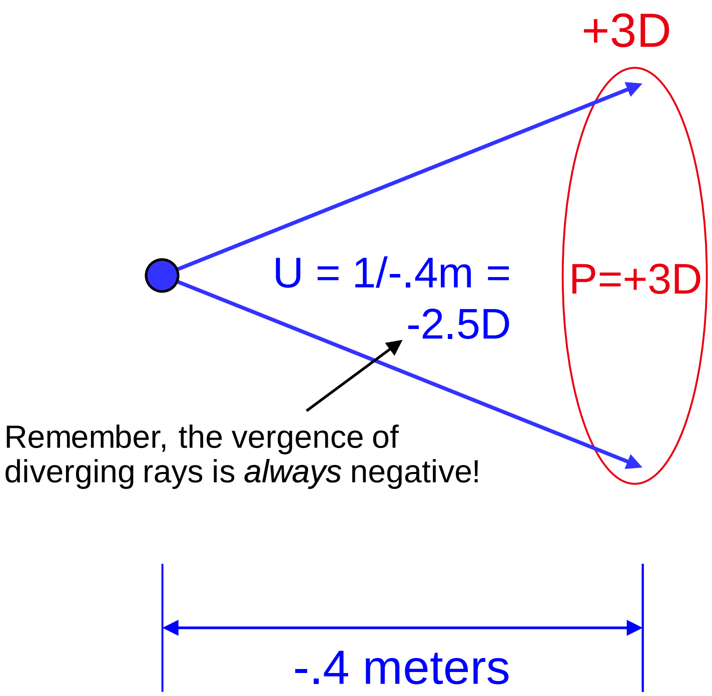


To determine the vergence ***U*** of the incoming light, take the reciprocal of the distance from its source or focal point:
U = 1/-.4m = **-2.5D**

Vergence: The Vergence Formula

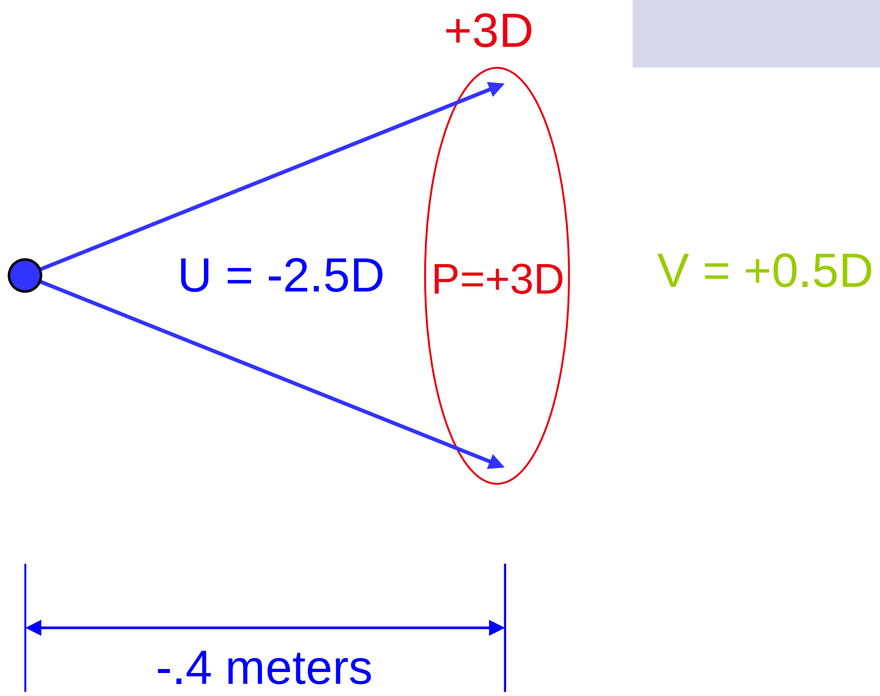
U + P = V

Plugging these values into the Vergence Formula:

U + P = V

-2.5 + (+3) = V

+0.5 = V



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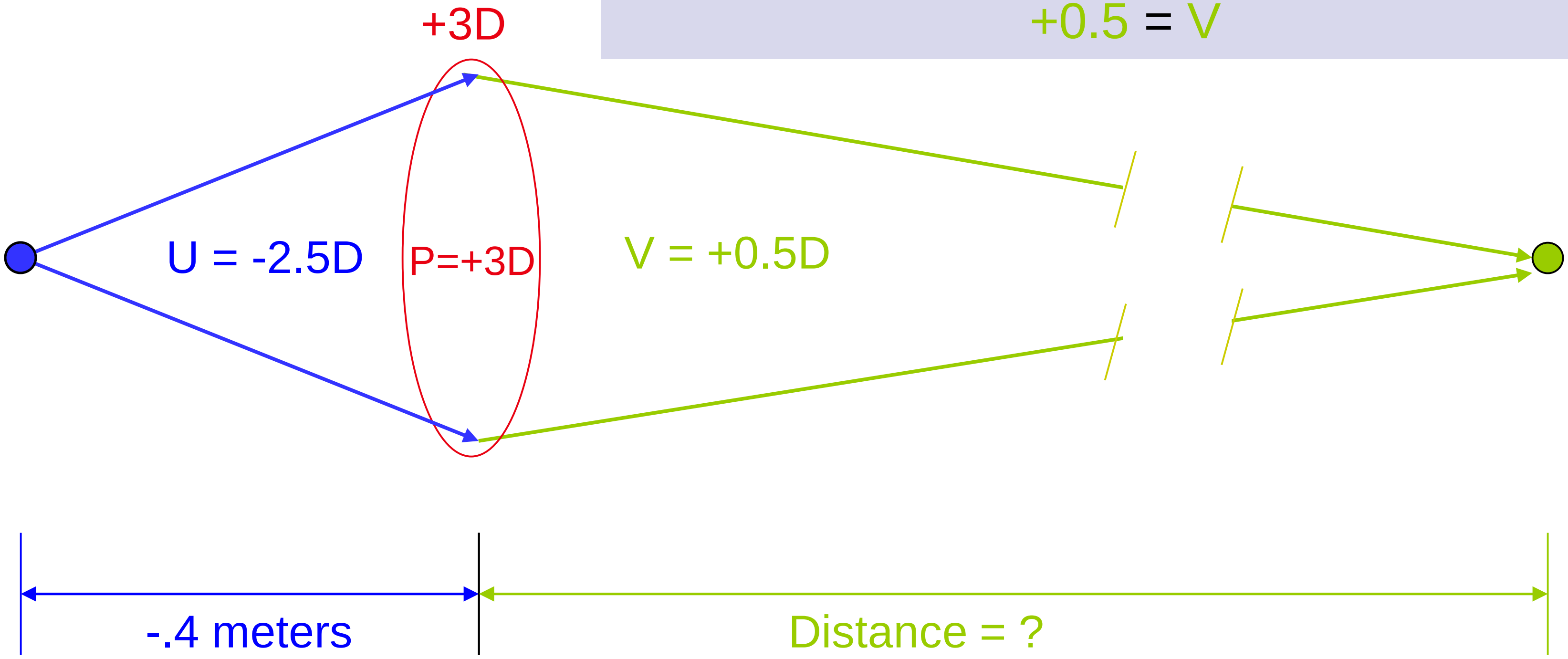

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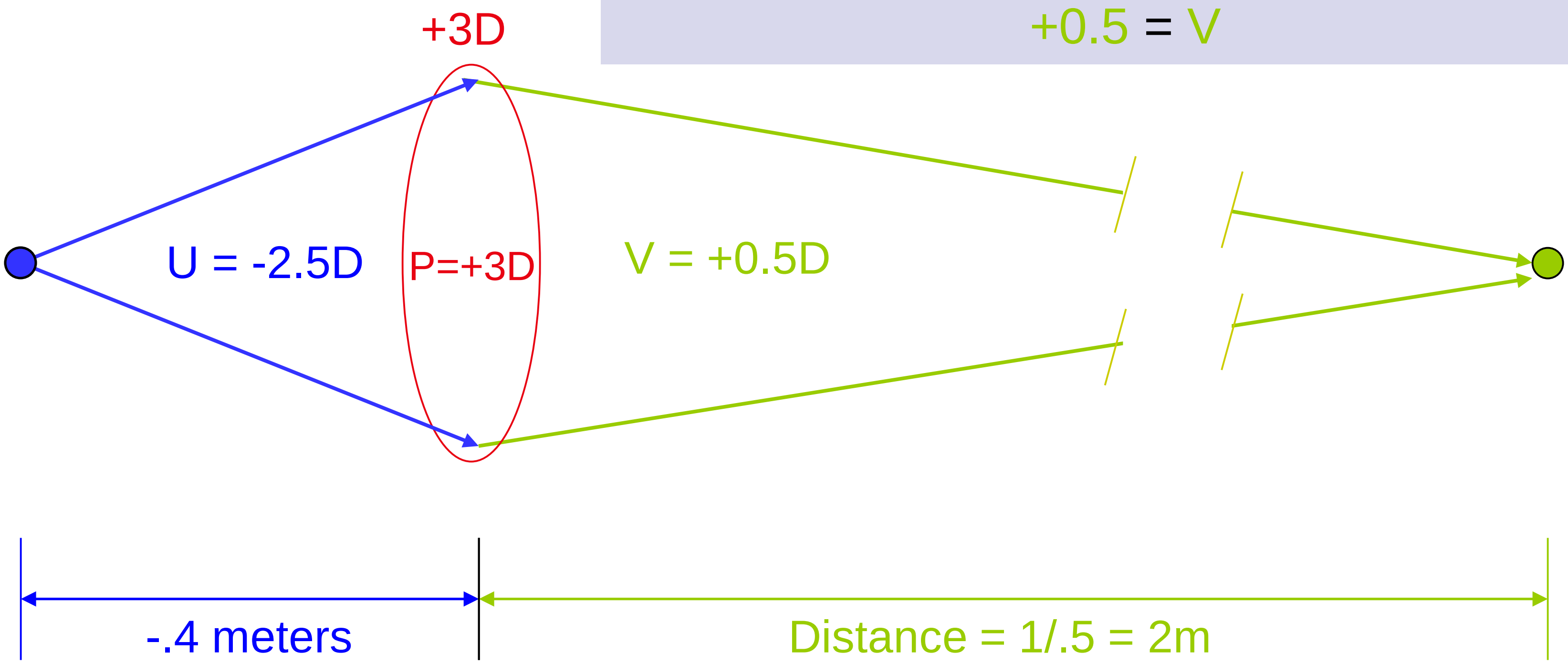

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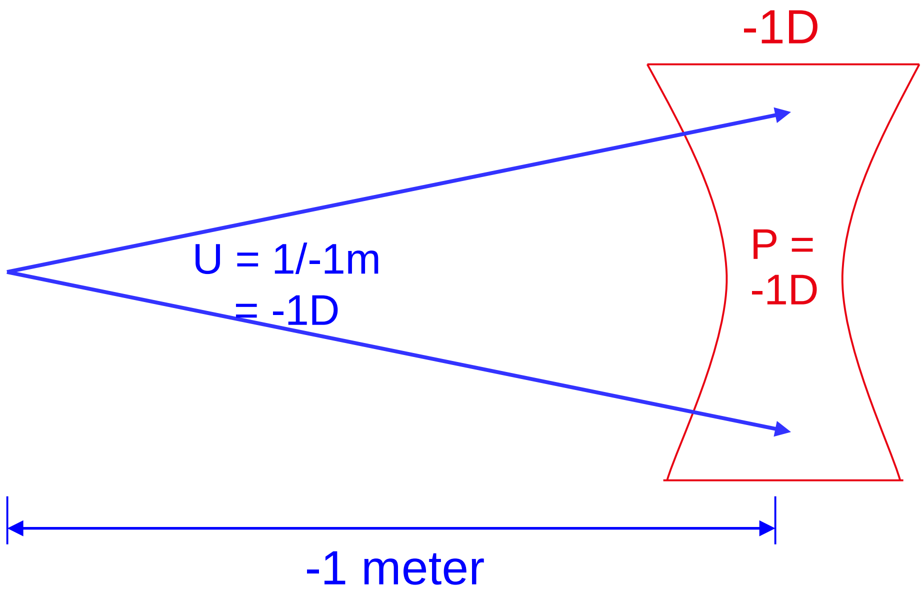

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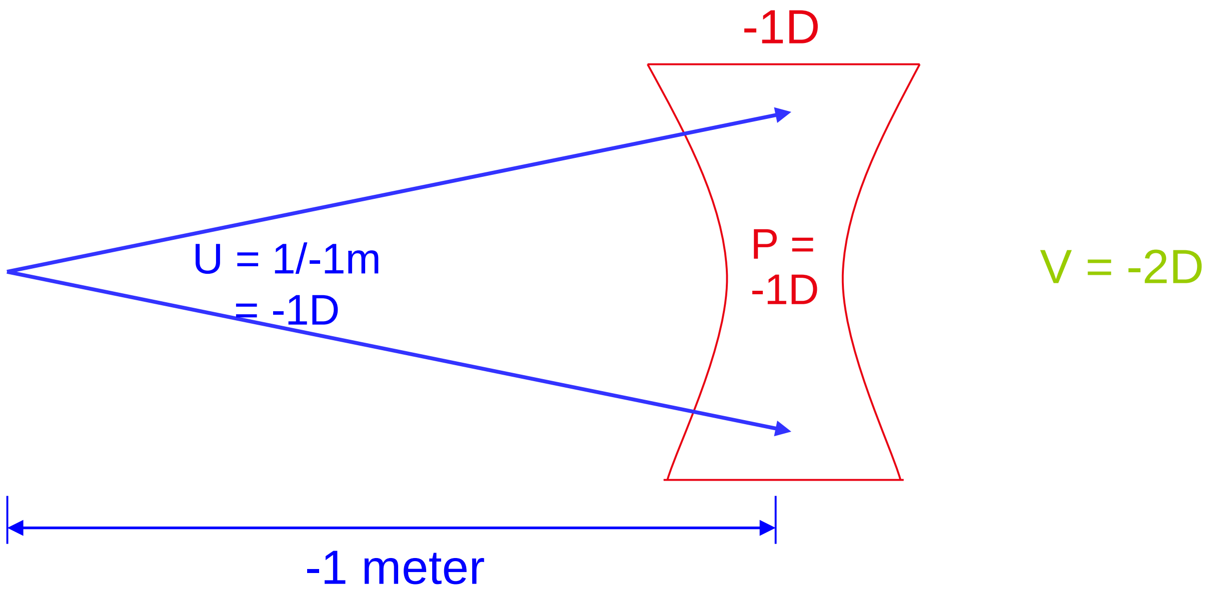

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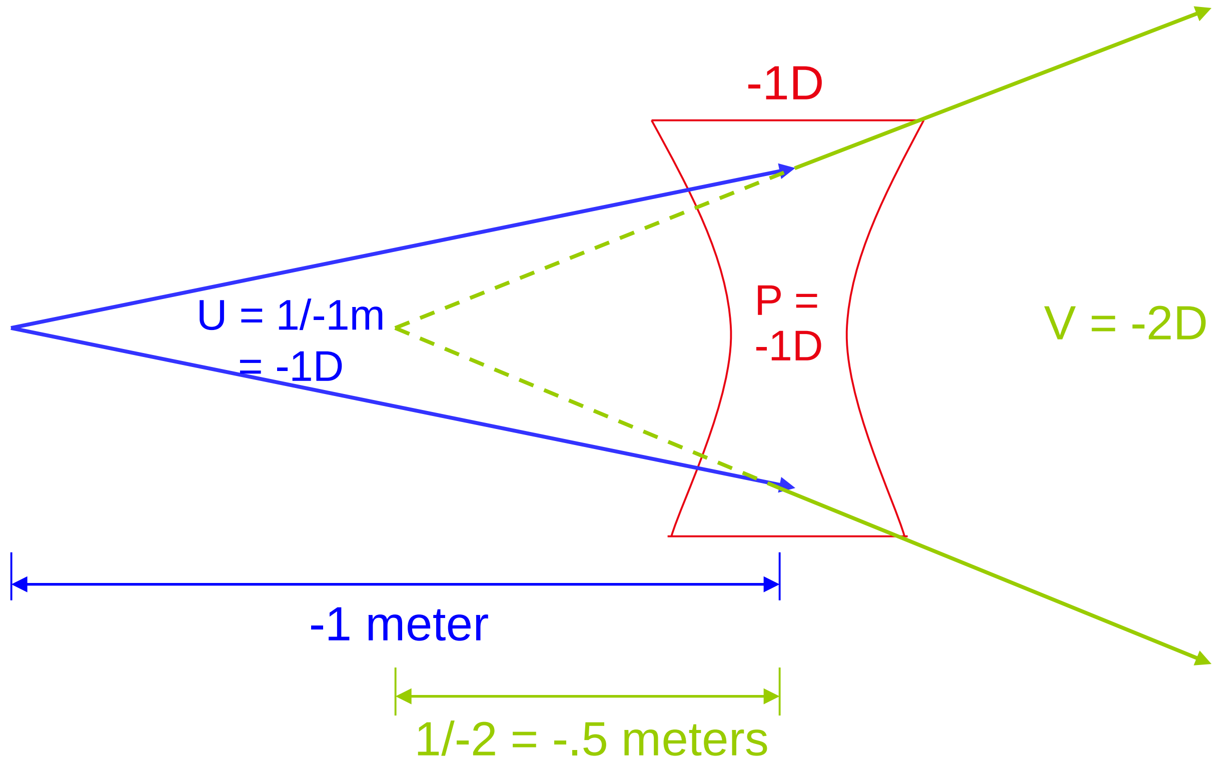

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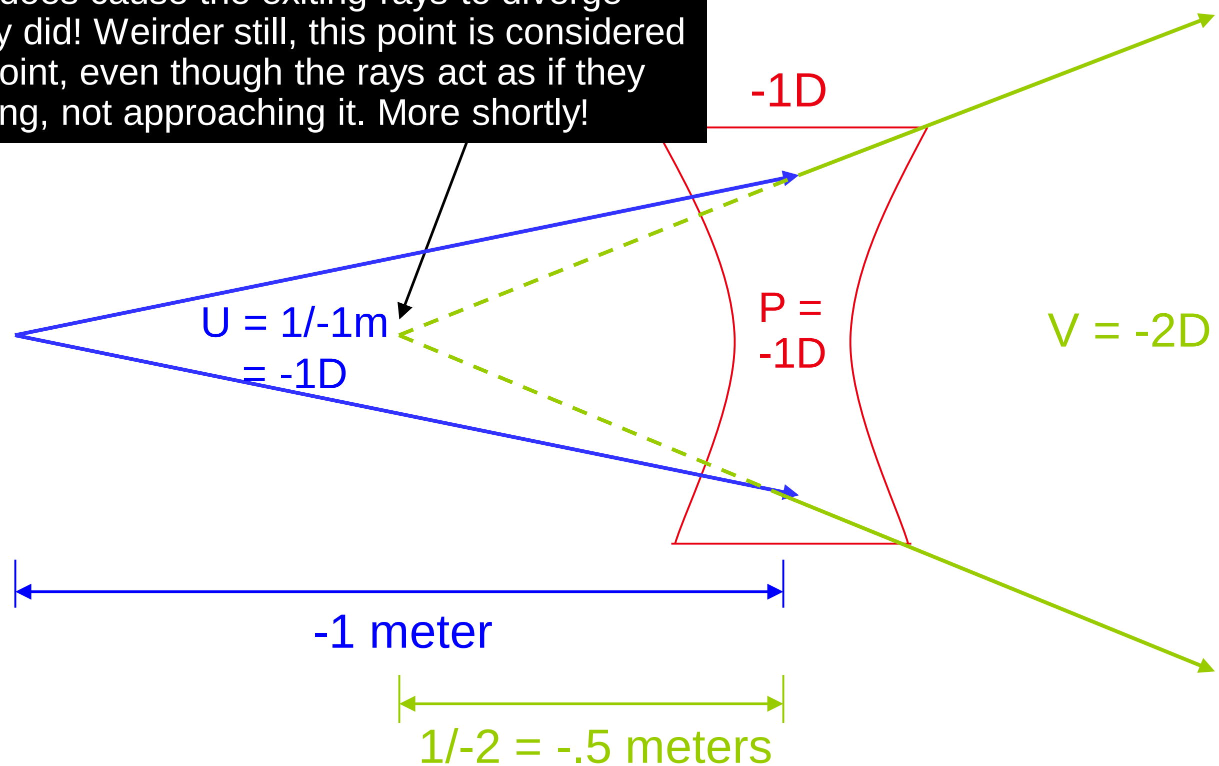
Note that the cartoon seems to indicate that the lens causes the rays to originate from this point. This of course is **not** what happens. Nonetheless, the lens does cause the exiting rays to diverge ***as if*** they did! Weirder still, this point is considered a focal point, even though the rays act as if they are leaving, not approaching it. More shortly!



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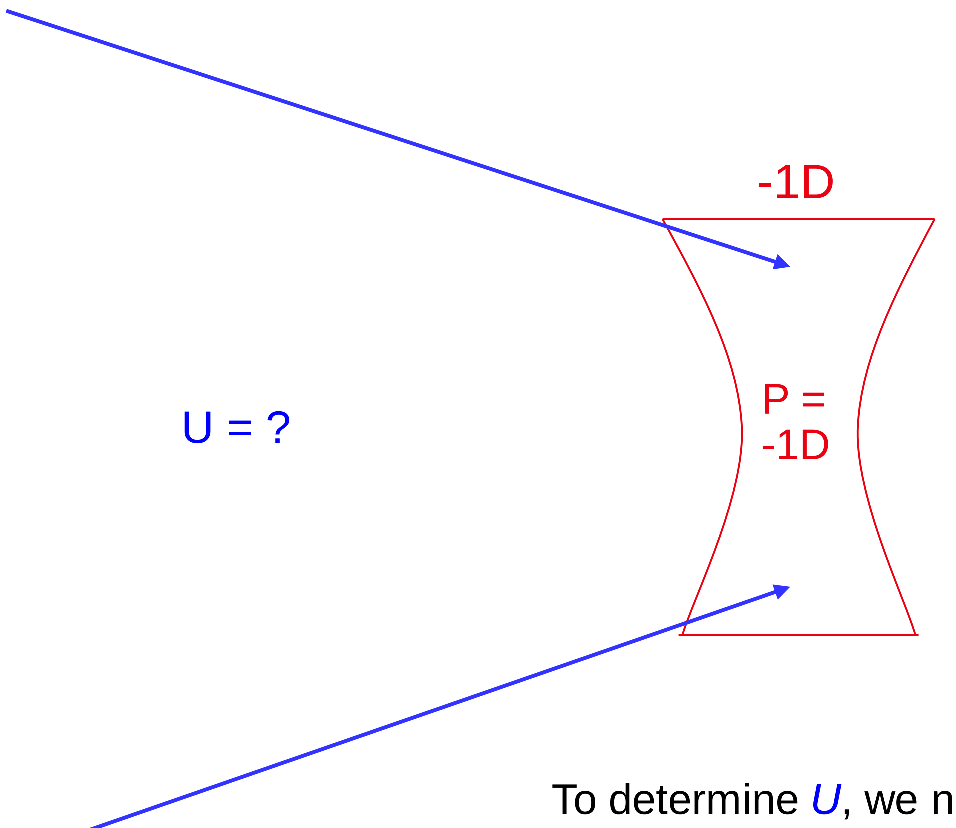

To determine *U*, we need to know…

Vergence: The Vergence Formula

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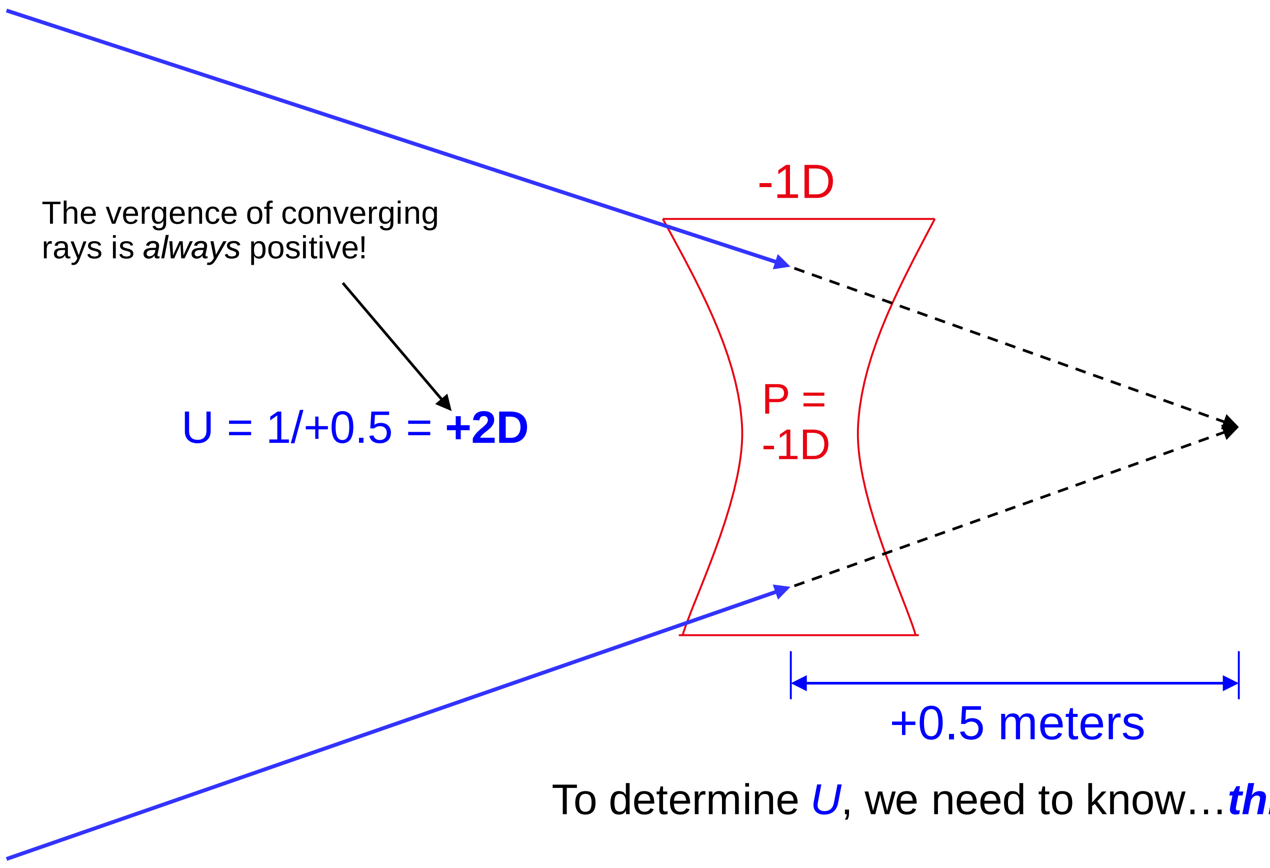

To determine *U*, we need to know…***this*** distance.

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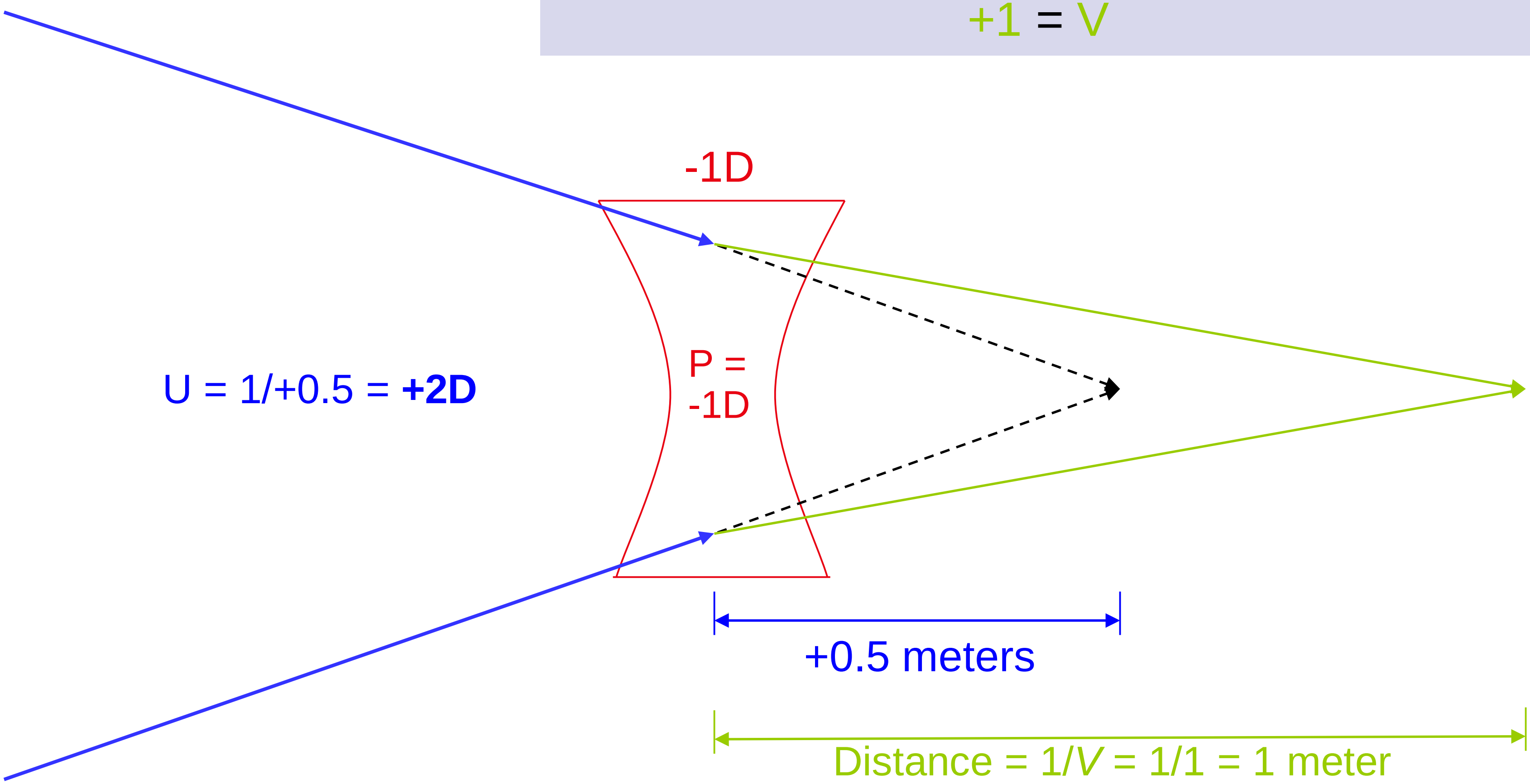

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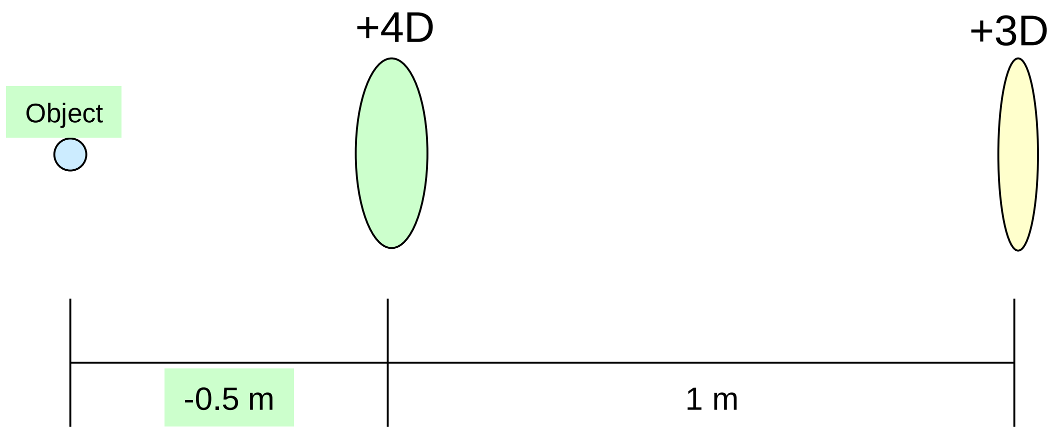
U + P = V



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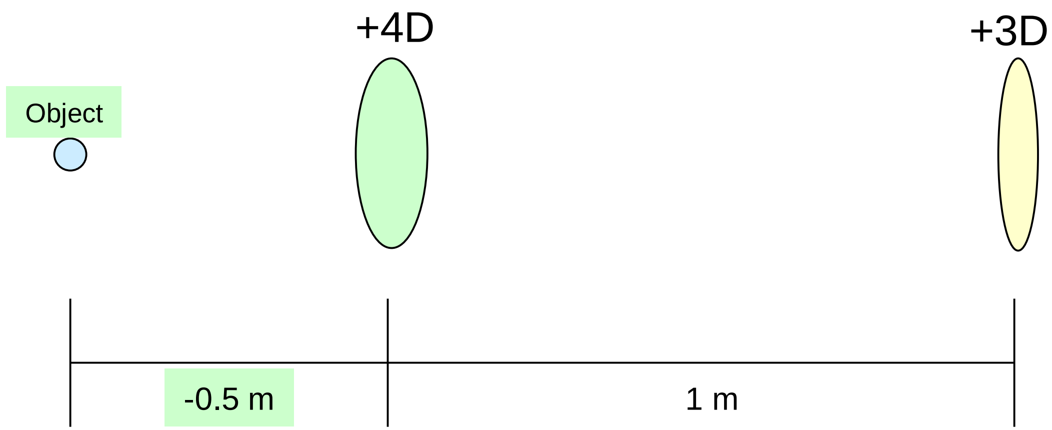

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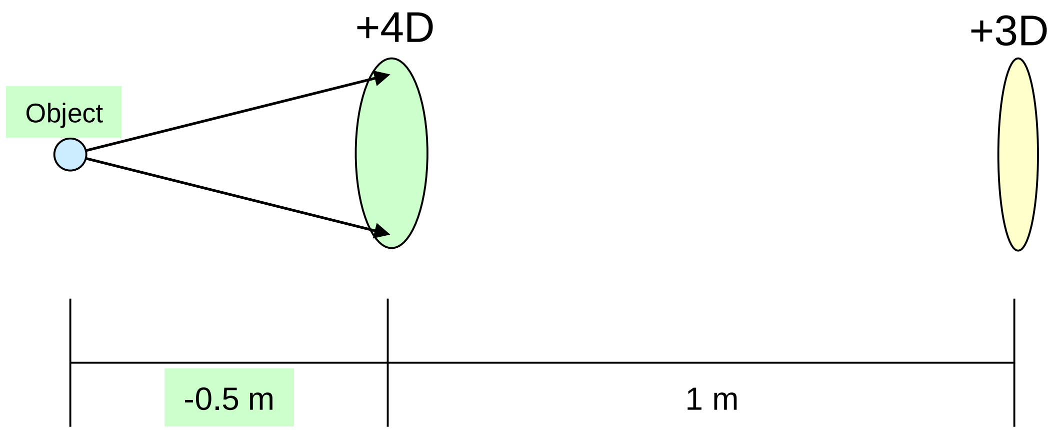

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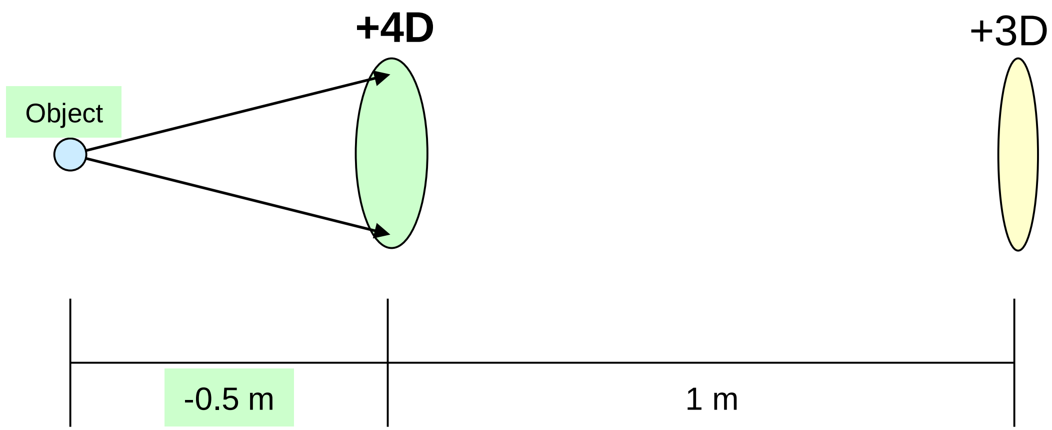

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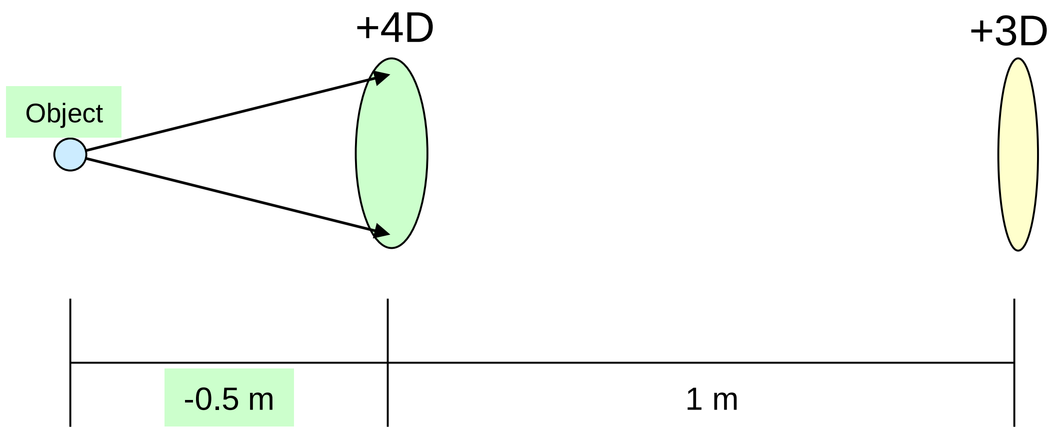

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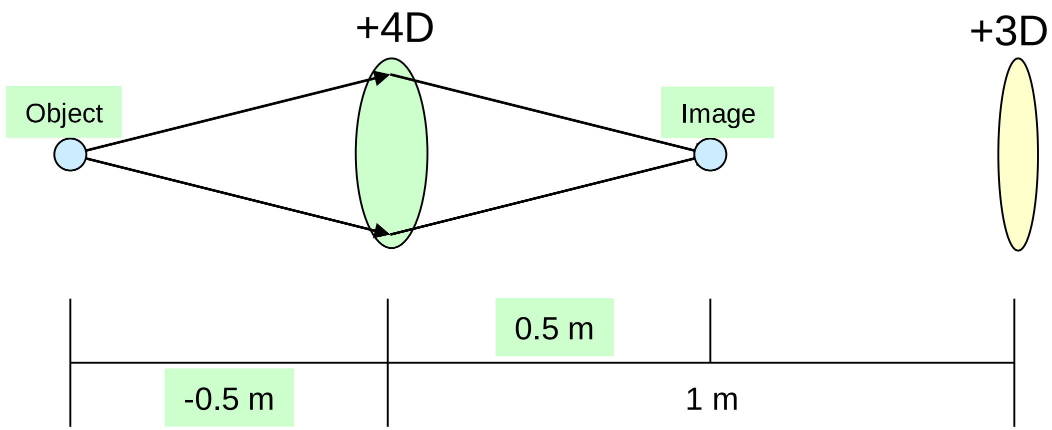

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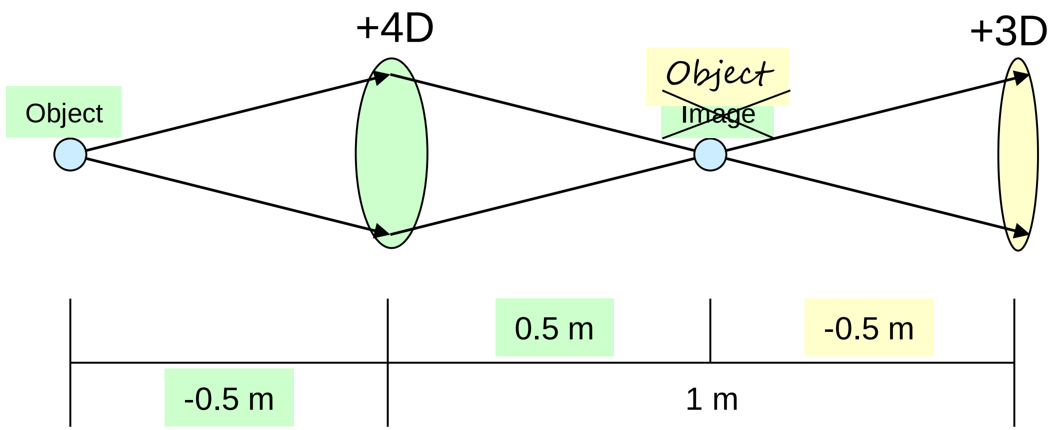

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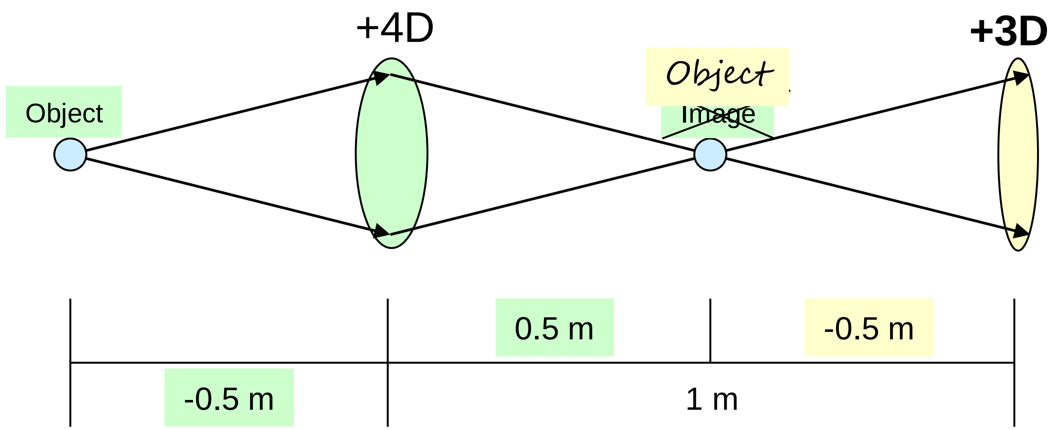

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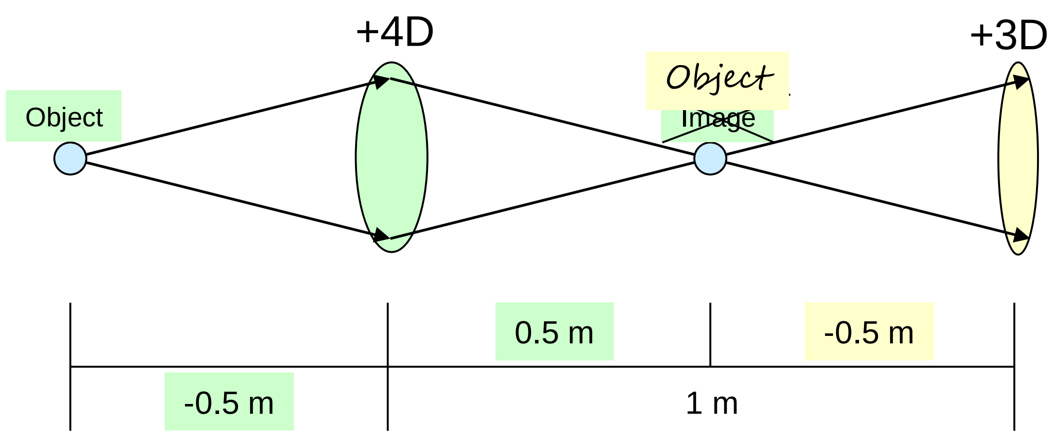

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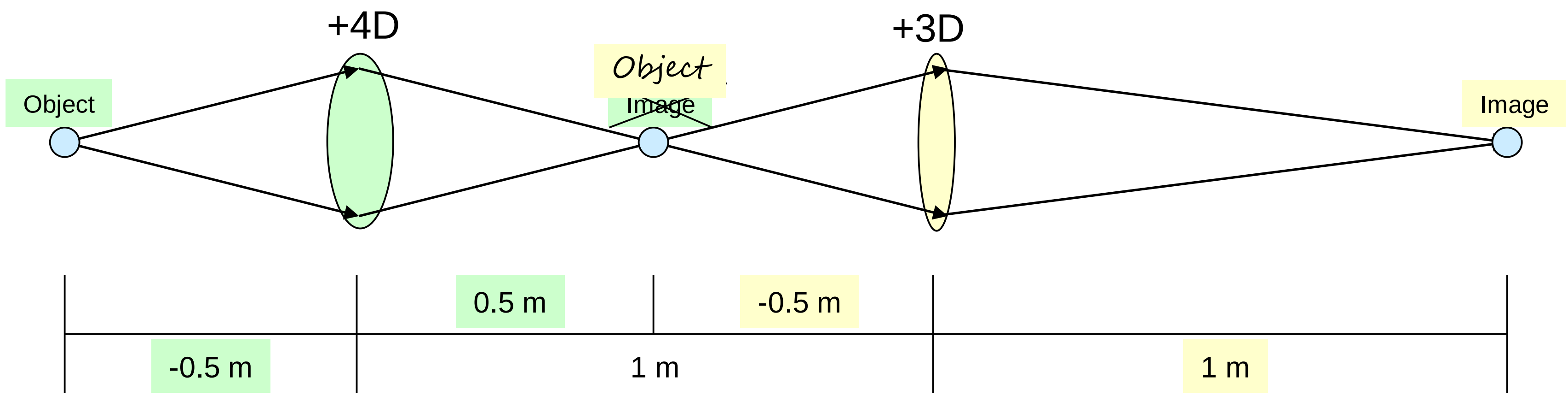
The image from the first lens is 1/2 = .5 m to the right of the first lens

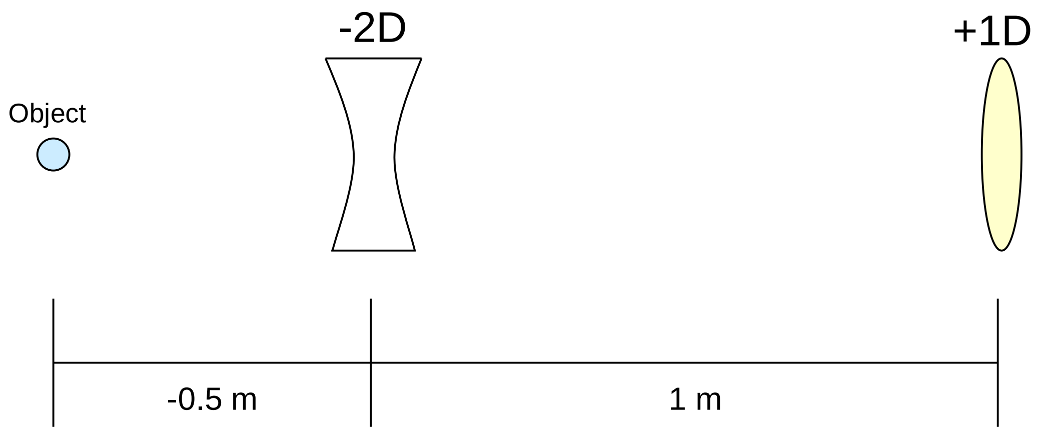
For the +3D lens:
U = 1/-0.5 = -2D
P = +3D
V = -2 + (+3) = +1D
The image formed by the second lens is 1/1 = **1 m to the right of the second lens**



Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

U + P = V

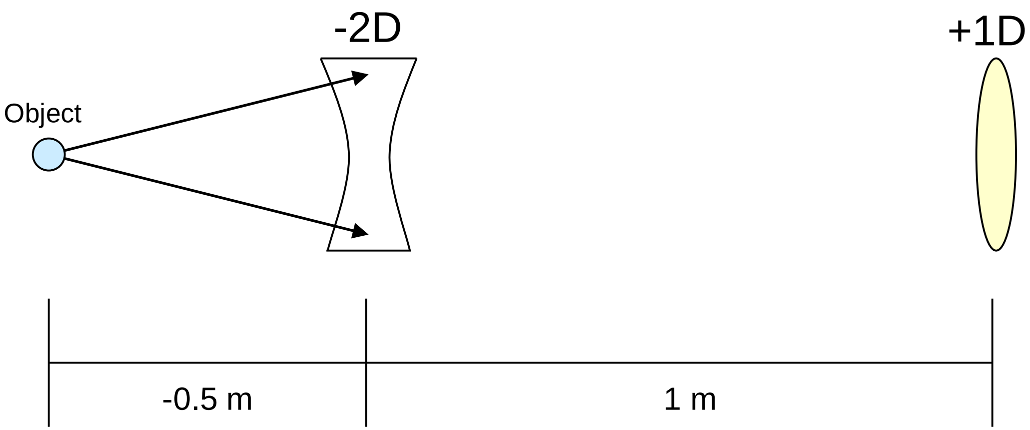


Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

$$U + P = V$$

For the -2D lens:
U = 1/-0.5 = -2D

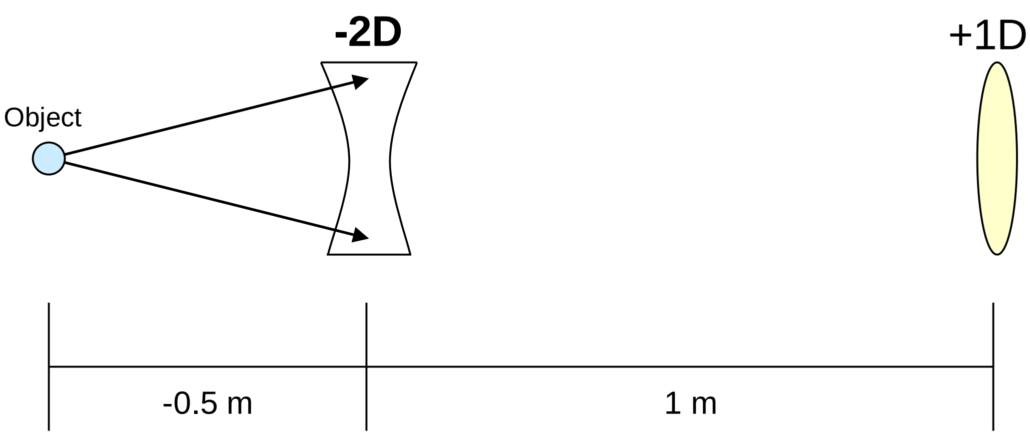


Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

U + P = V

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D

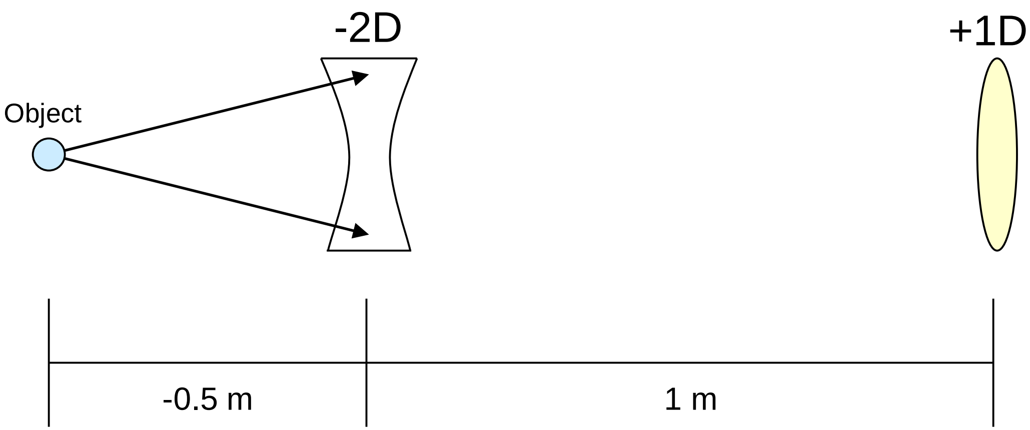


Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

U + P = V

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D
V = -2 + (-2) = -4D

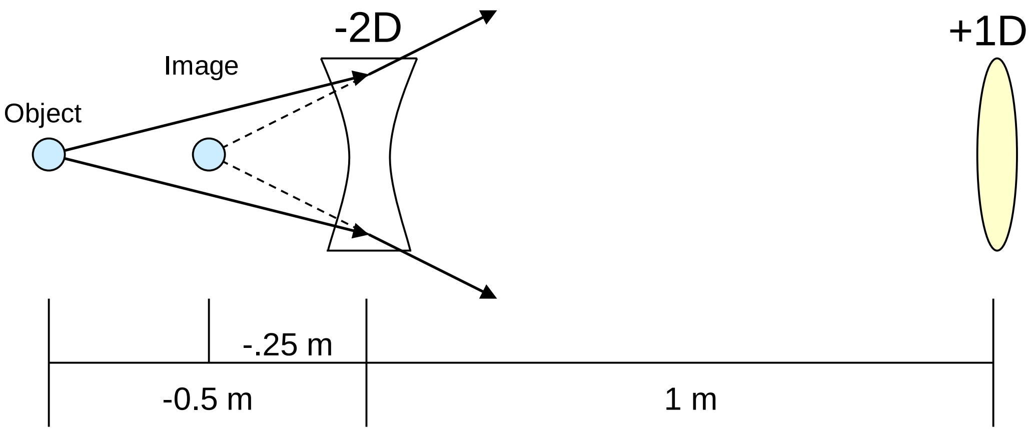


Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

U + P = V

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D
V = -2 + (-2) = -4D
The image from the first lens is 1/-4 = .25 m to the left of the first lens



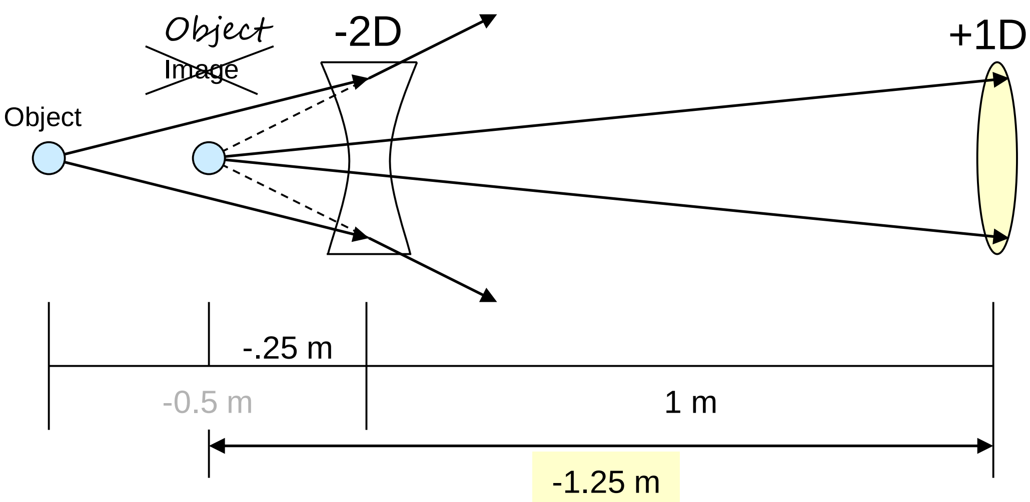
Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

U + P = V

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D
V = -4D
The image from the first lens is 1/-4 = .25 m to the left of the first lens

For the +1D lens:
U = 1/-1.25 m = -0.8D



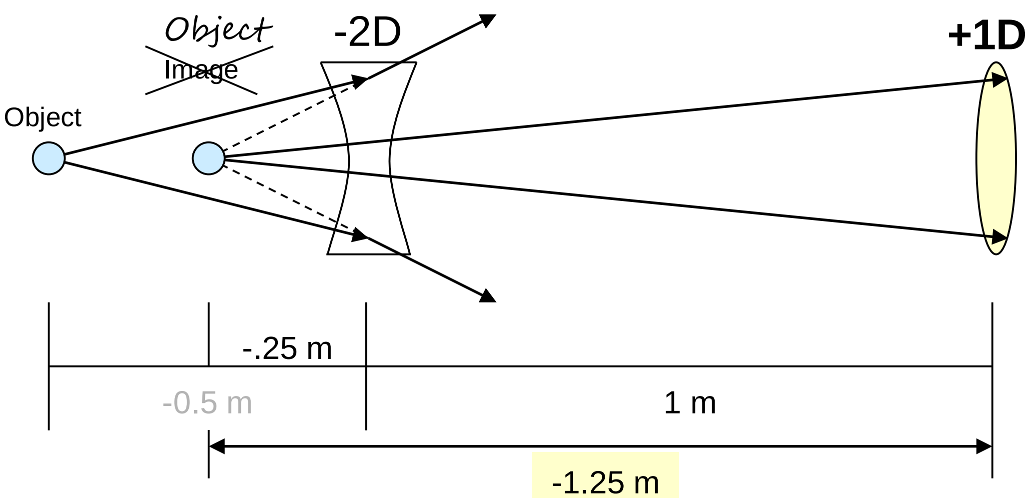
Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

U + P = V

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D
V = -4D
The image from the first lens is 1/-4 = .25 m to the left of the first lens

For the +1D lens:
U = 1/-1.25 m = -0.8D
P = +1D



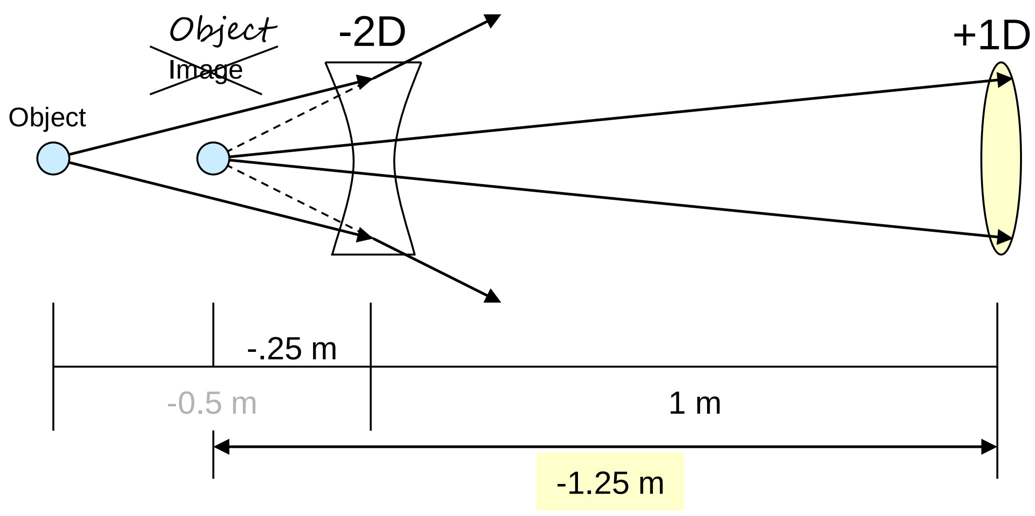
Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

U + P = V

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D
V = -4D
The image from the first lens is 1/-4 = .25 m to the left of the first lens

For the +1D lens:
U = 1/-1.25 m = -0.8D
P = +1D
V = -0.8 + (+1) = +0.2D



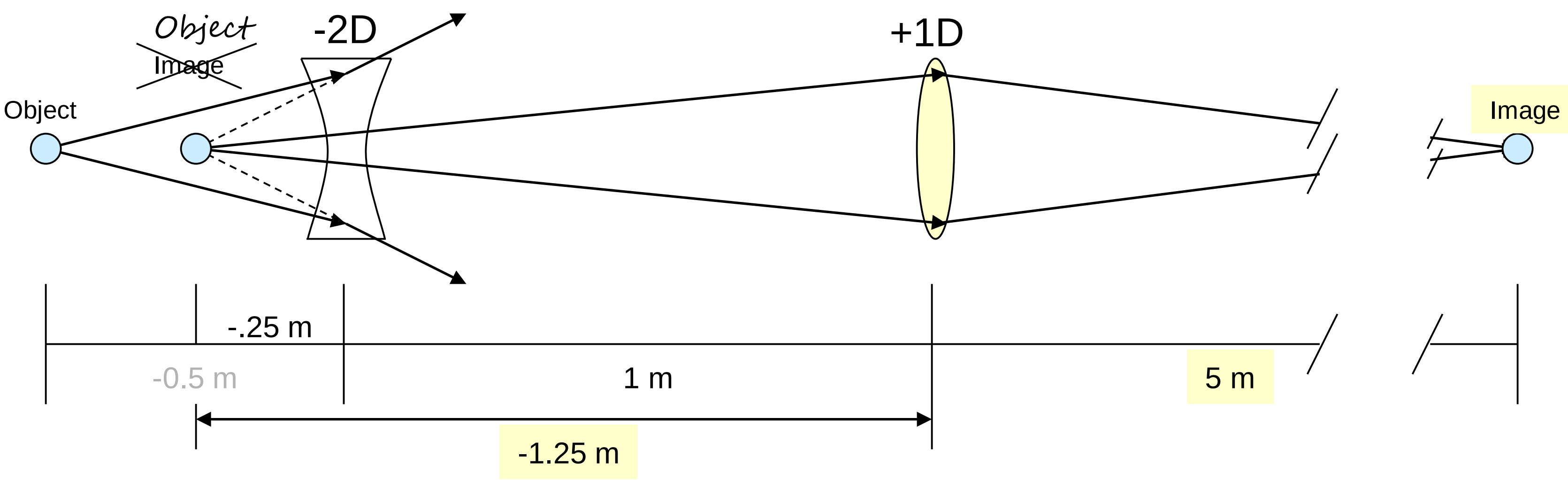
Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

U + P = V

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D
V = -4D
The image from the first lens is 1/-4 = .25 m to the left of the first lens

For the +1D lens:
U = 1/-1.25 m = -0.8D
P = +1D
V = -0.8 + (+1) = +0.2D
The image formed by the second lens is 1/+0.2 = 5 m to the right of the second lens



Vergence: T

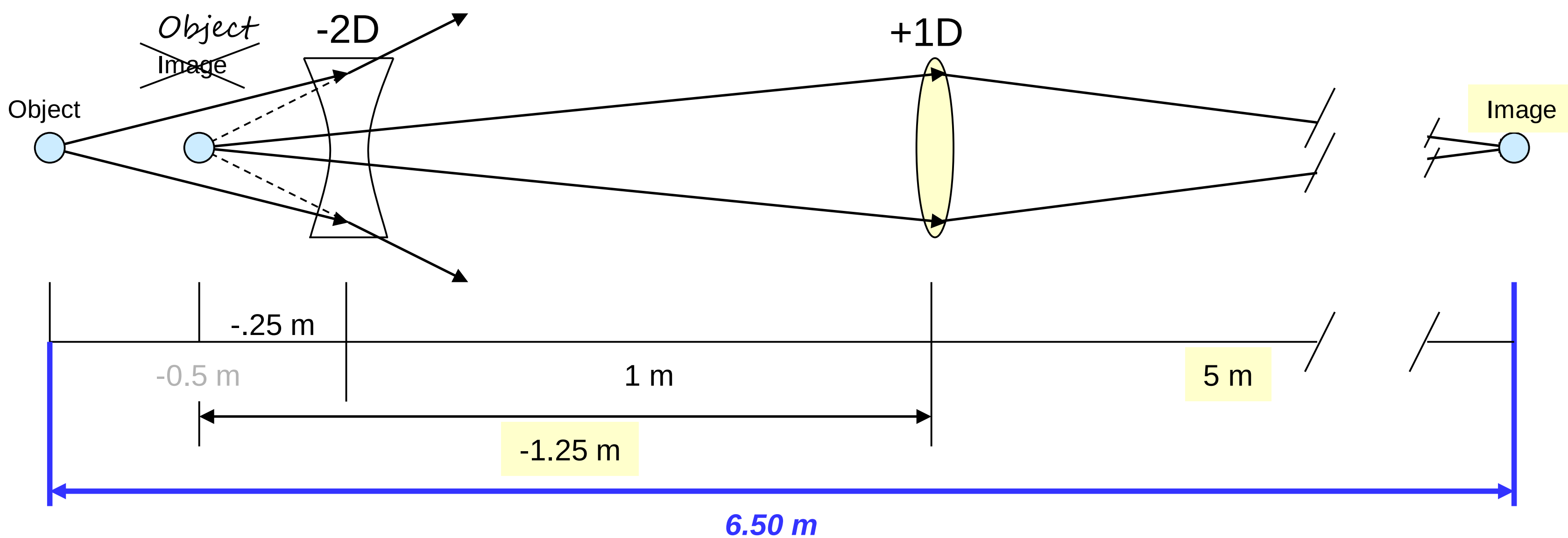
An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?
6.50 m

U + P = V

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D
V = -4D
The image from the first lens is 1/-4 = .25 m to the left of the first lens

For the +1D lens:
U = 1/-1.25 m = -0.8D
P = +1D
V = -0.8 + (+1) = +0.2D
The image formed by the second lens is 1/+0.2 = 5 m to the right of the second lens

Distance from object to image = 6.50 m



Vergence: T

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?
6.50 m

U + P = V

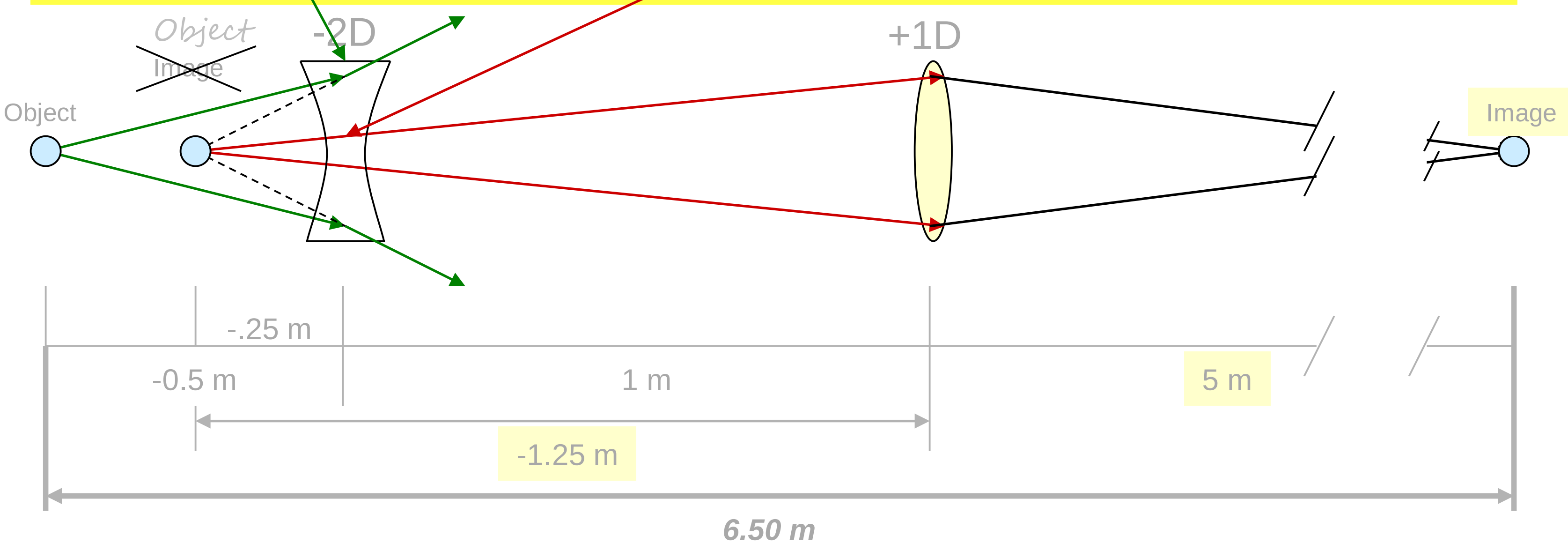
For
U =
P =
V =
The
= .2

m object
.50 m

But it looks like the rays from the first image/second object have to pass through the -2D lens ***twice*** *before reaching the +1D lens. Doesn't this refract those rays again?*

First pass through the lens

Second pass through the lens?



Vergence: T

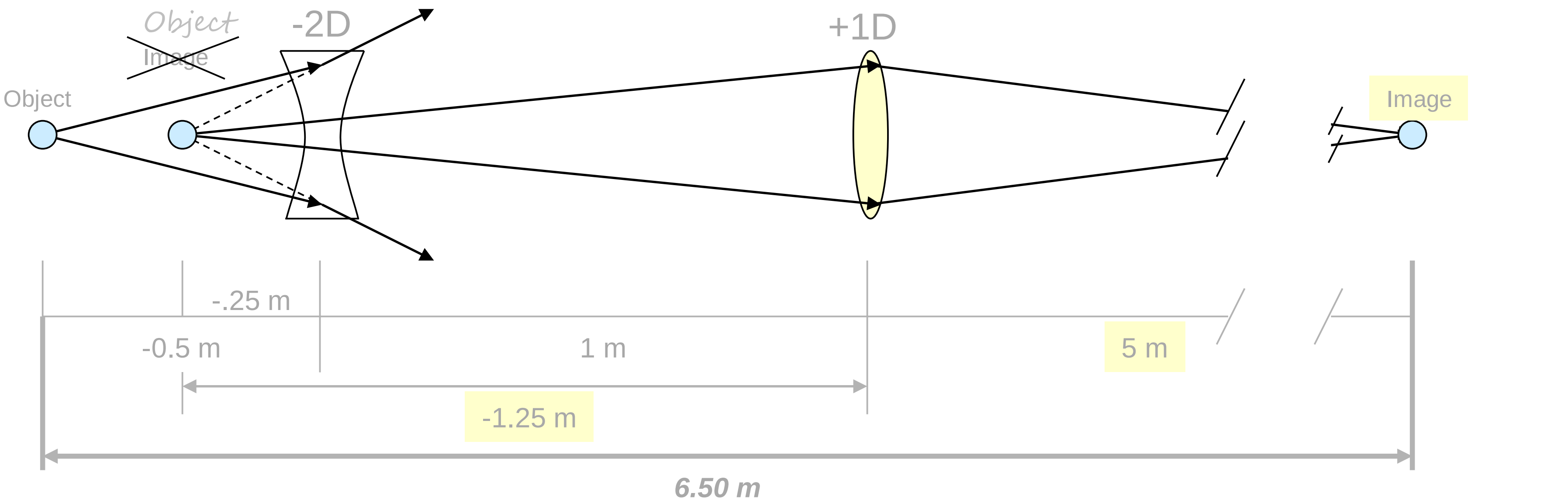
An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?
6.50 m

U + P = V

For
U =
P =
V =
The
= .2

m object
.50 m

But it looks like the rays from the first image/second object have to pass through the -2D lens ***twice*** *before reaching the +1D lens. Doesn't this refract those rays again?*
NO. After the light bouncing off the physical object passes through the -2D lens, it acts *as if* it came from the first image/second object location, but it's not *really* coming from there. In fact, as we will come to see, clinical optics is less a description of what light actually does than it is a powerful **metaphor** that allows us to make useful descriptions (and prescriptions!) of what light does. In essence, clinical optics is a **convenient fiction**. More on this (much) later.



Vergence: T

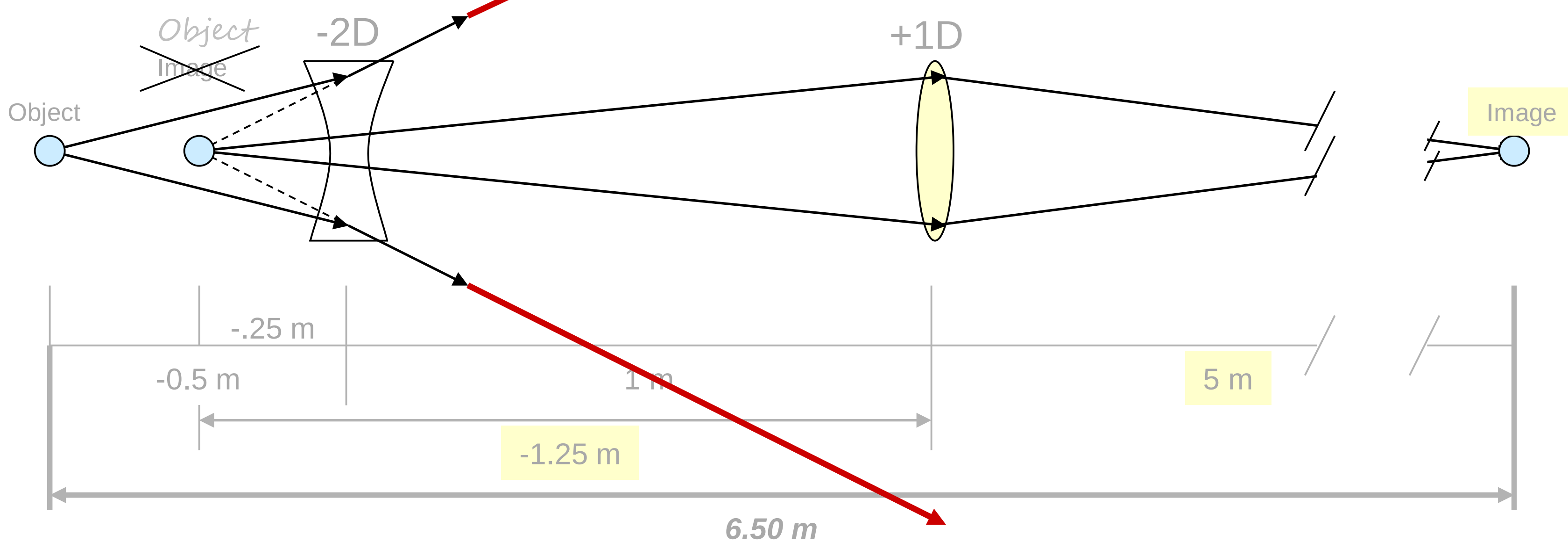
An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

OK, but there's another problem. Clearly, the rays that have passed through the -2D are far too divergent to pass through the +1D lens—they're going to miss it by a mile! How can these rays possibly be refracted by the second lens?

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D
V = -4D
The image from the first lens is 1/-4 = .25 m to the left of the first lens

For the +1D lens:
U = 1/-1.25 m = -0.8D
P = +1D
V = +0.2D
The image from the second lens is 1/+0.2 = 5 m to the right of the second lens

Distance from object to image = 6.50 m



Vergence: T

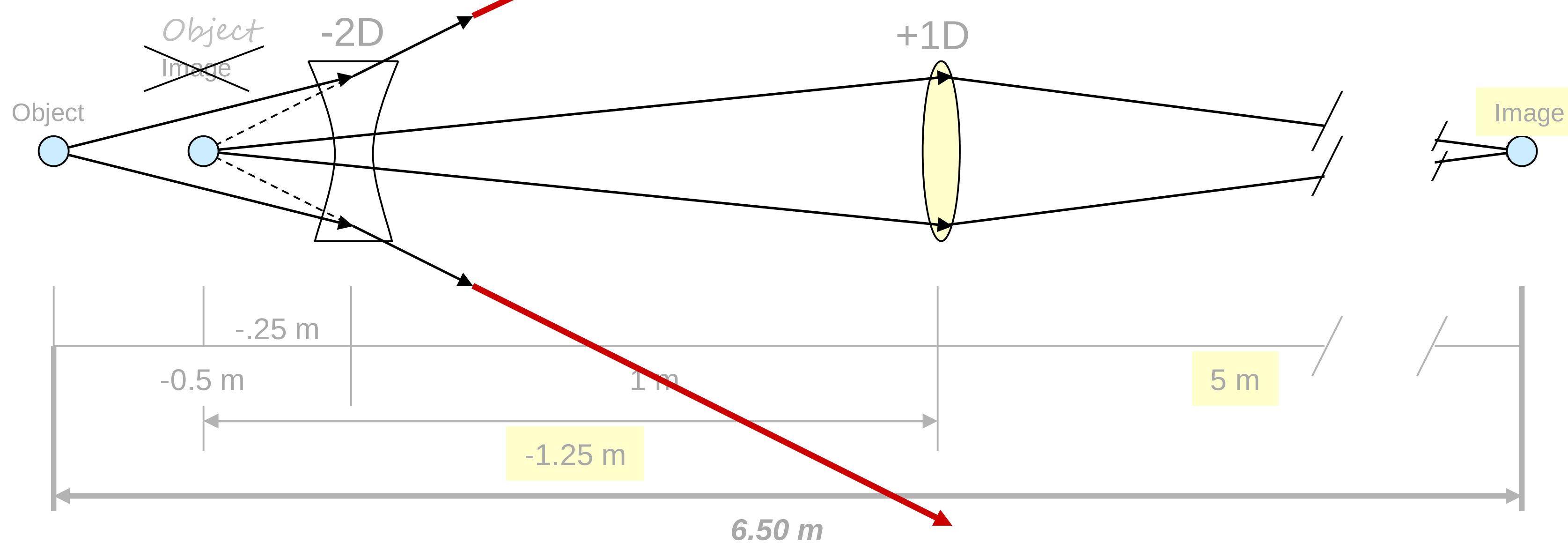
An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

OK, but there's another problem. Clearly, the rays that have passed through the -2D are far too divergent to pass through the +1D lens—they're going to miss it by a mile! How can these rays possibly be refracted by the second lens?
Don't let such 'drawing artifacts' fool you—some of the light will make it through the second lens.

For the -2D lens:
U = 1/-0.5 = -2D
P = -2D
V = -4D
The image from the first lens is 1/-4 = .25 m to the left of the first lens

For the +1D lens:
U = 1/-1.25 m = -0.8D
P = +1D
V = +0.2D
The image from the second lens is 1/+0.2 = 5 m to the right of the second lens

Distance from object to image = 6.50 m



Vergence: T…

An object is located ½ m to the left of a -2D lens, which is in turn 1 m to the left of a +1D lens. How far is the final image from the object?

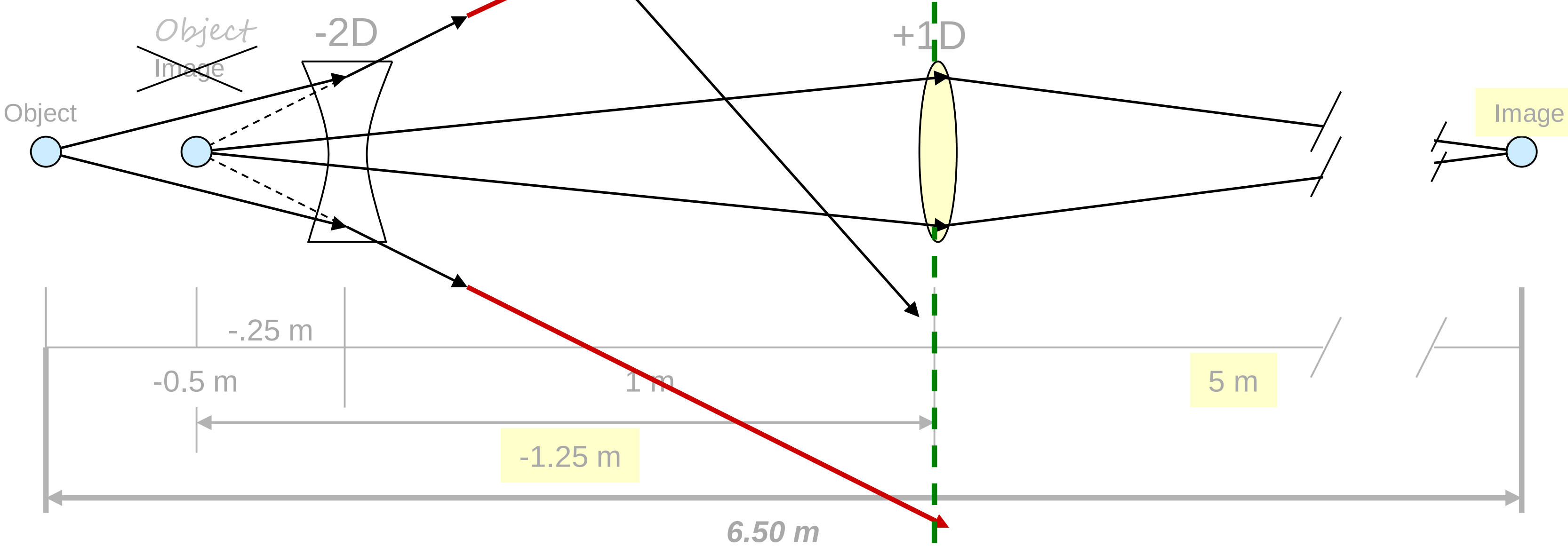
OK, but there's another problem. Clearly, the rays that have passed through the -2D are far too divergent to pass through the +1D lens—they're going to miss it by a mile. How can these rays possibly be refracted by the second lens?
Don't let such 'drawing artifacts' fool you—some of the light will make it through the second lens.

In fact, to make this point explicitly, illustrators will sometimes use dashed lines to 'extend' a lens so as to capture the rays in question.

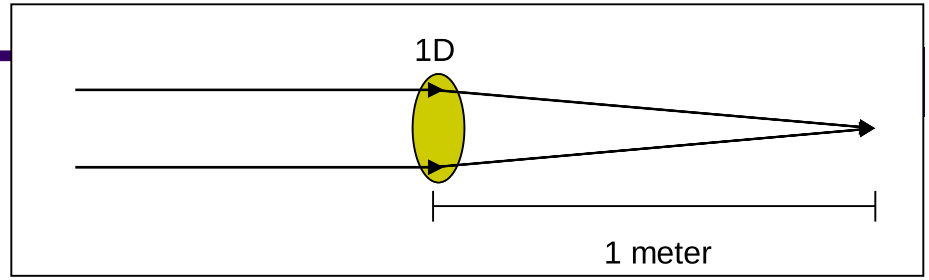
For
U =
P = -2D
V = -4D
The image from the first lens is 1/-4 = .25 m to the left of the first lens

lens:
m = -0.8D
P = +1D
V = +0.2D
The image from the second lens is 1/+0.2 = 5 m to the right of the second lens

Distance from object to image = 6.50 m



Vergence: T a

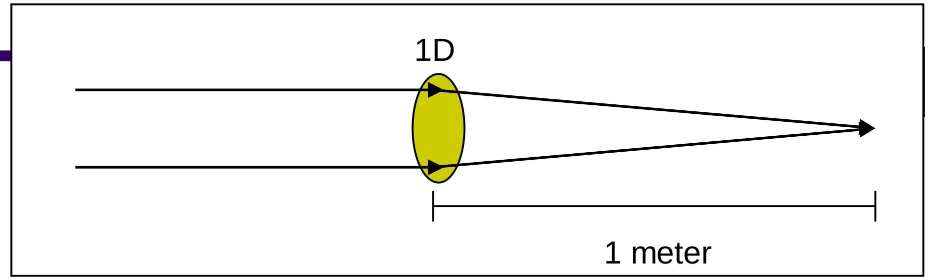


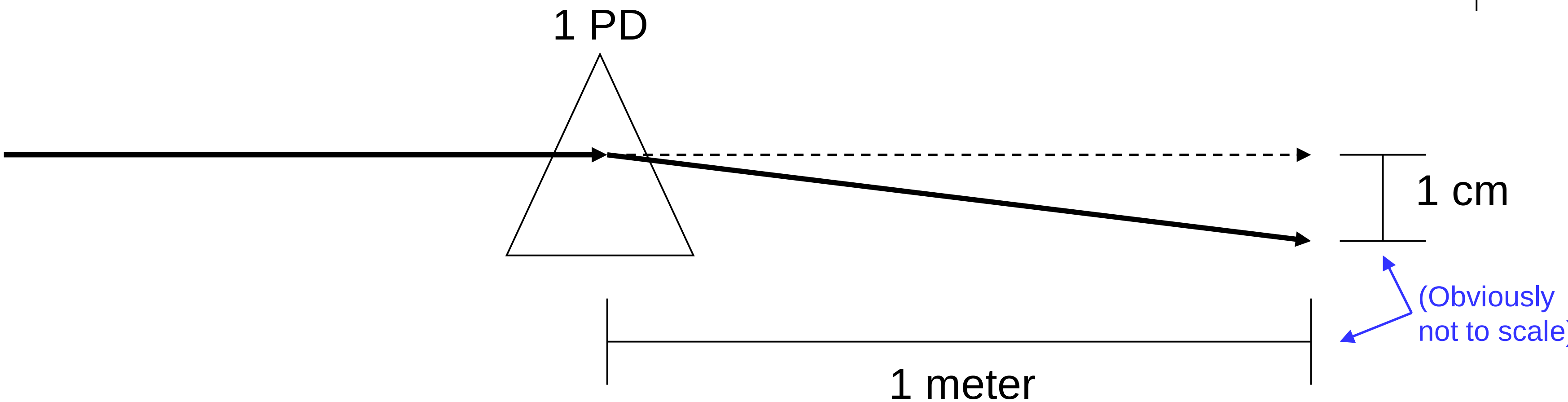
- The ability of a lens to induce vergence is expressed in diopters
  - Dioptric power of a lens: The reciprocal of the distance (in meters) to the point where incoming parallel light rays would intersect after passing through the lens

We encountered this slide a few minutes ago…

The notion that a diopter does something to light over the course of a meter should remind you of the effect a ***prism*** has on light…

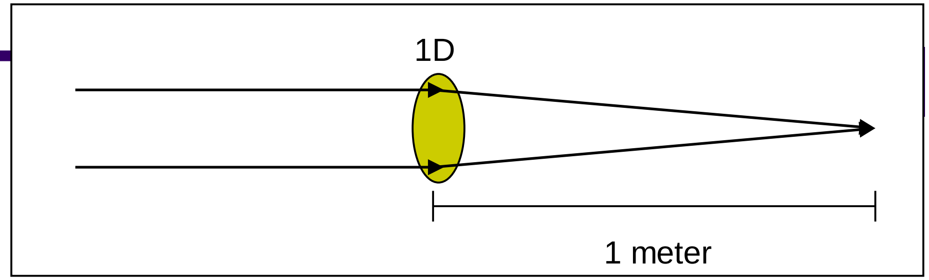
Vergence: T la

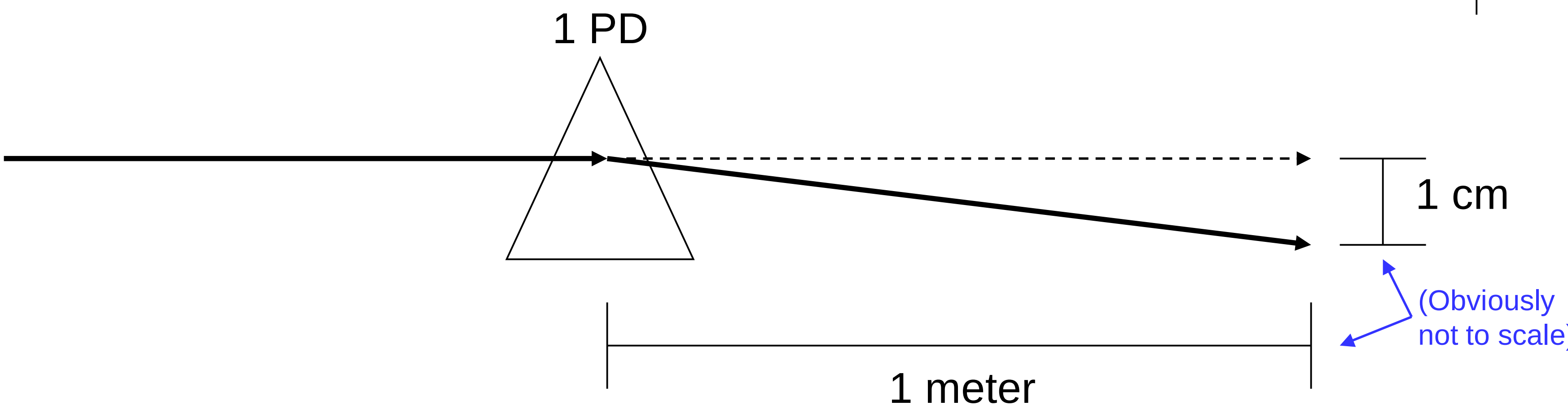




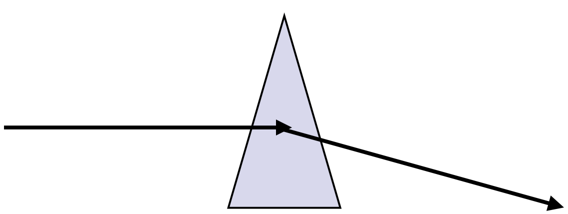
A prism diopter (**PD**, or Δ) displaces light 1 cm at 1 meter.

Vergence: T … a

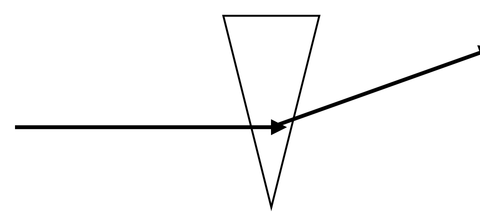




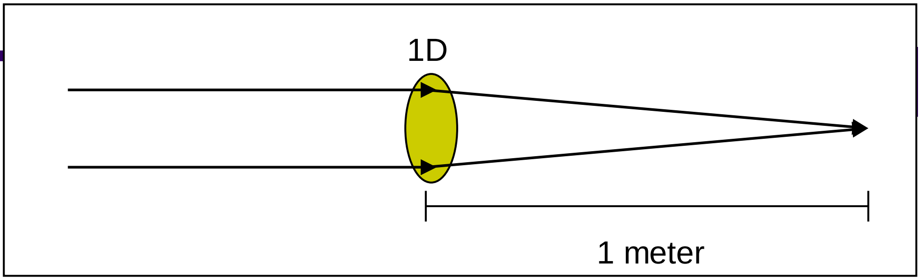
A prism diopter (**PD**, or Δ) displaces light 1 cm at 1 meter.

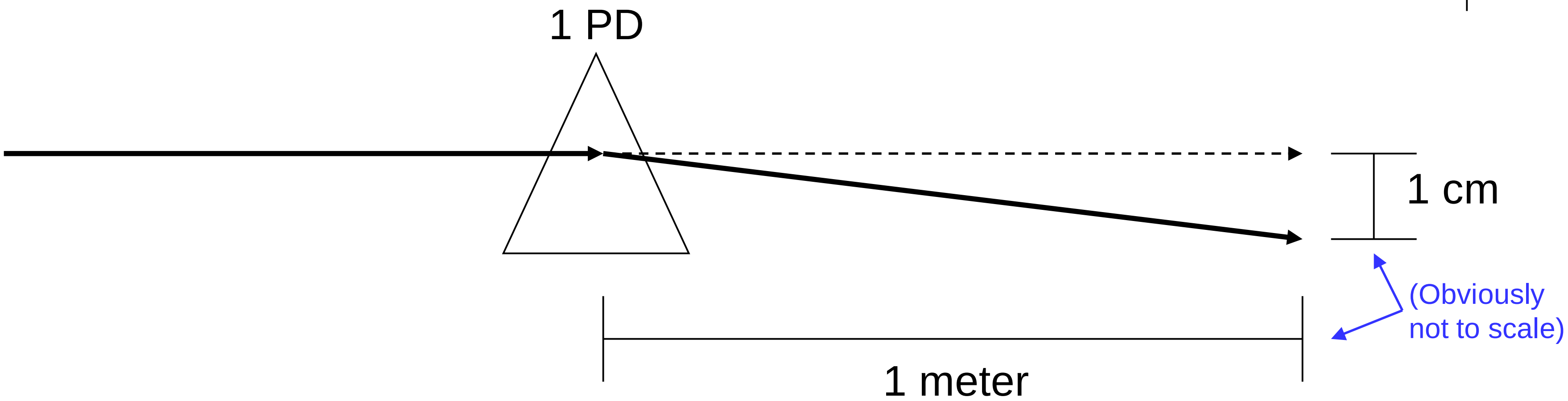


Which do prisms induce:
***con**vergence or **di**vergence?*

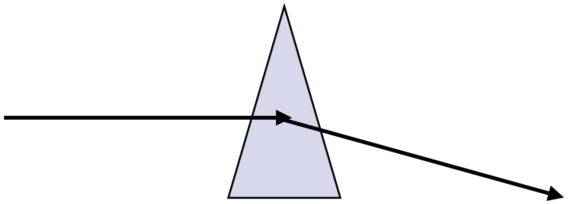


Vergence: T a

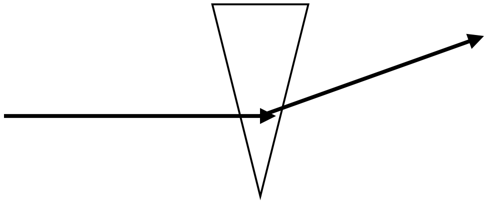




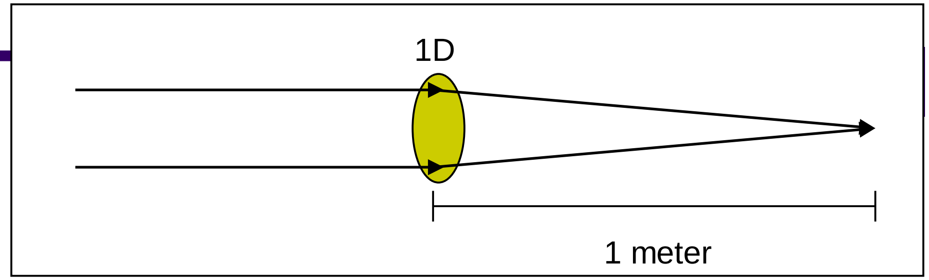
A prism diopter (**PD**, or Δ) displaces light 1 cm at 1 meter.

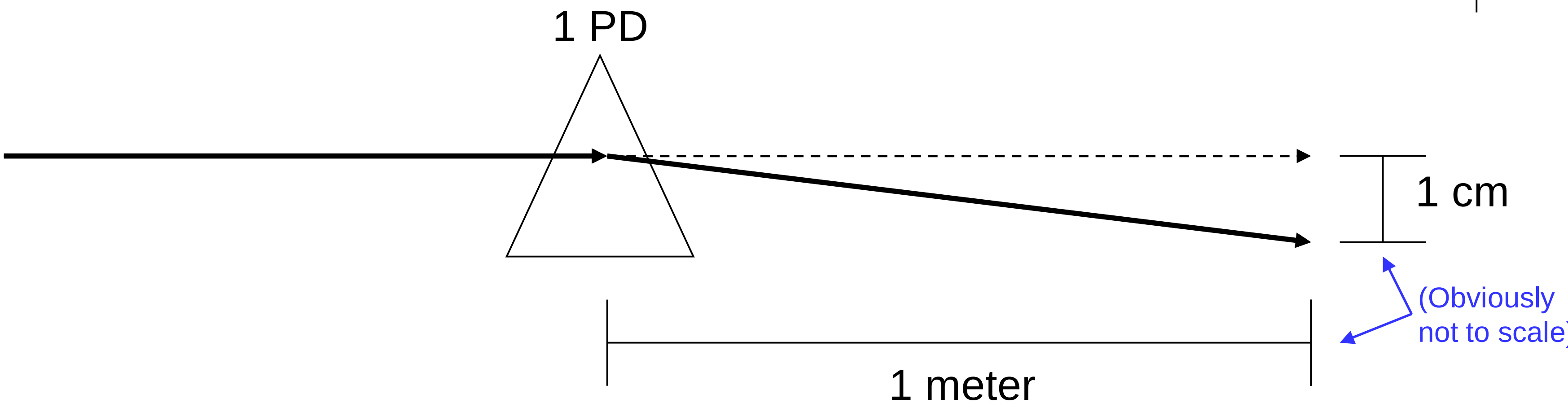


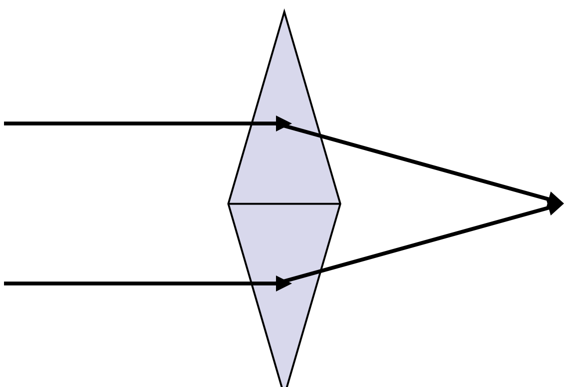
Which do prisms induce:
***con**vergence or **di**vergence?*
Neither--prisms do ***not*** induce vergence! Prisms cause light rays to *change direction*, but not to converge or diverge.



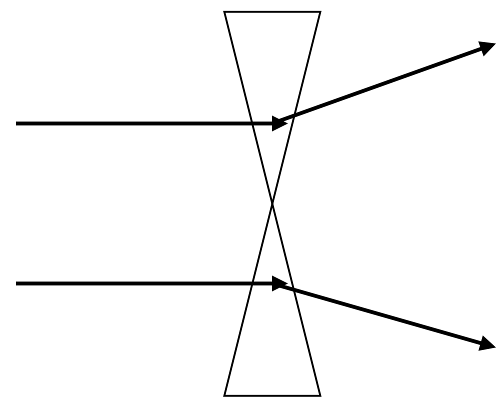
Vergence: T a



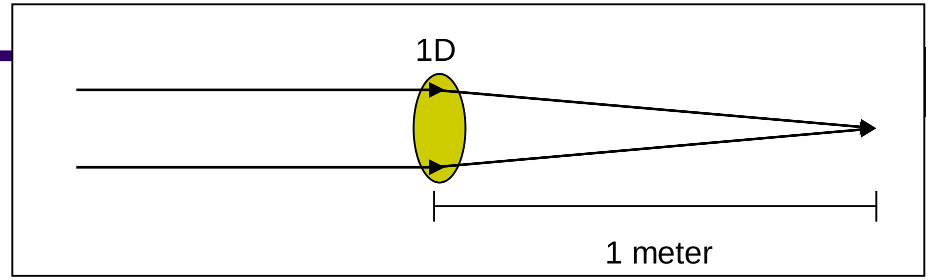


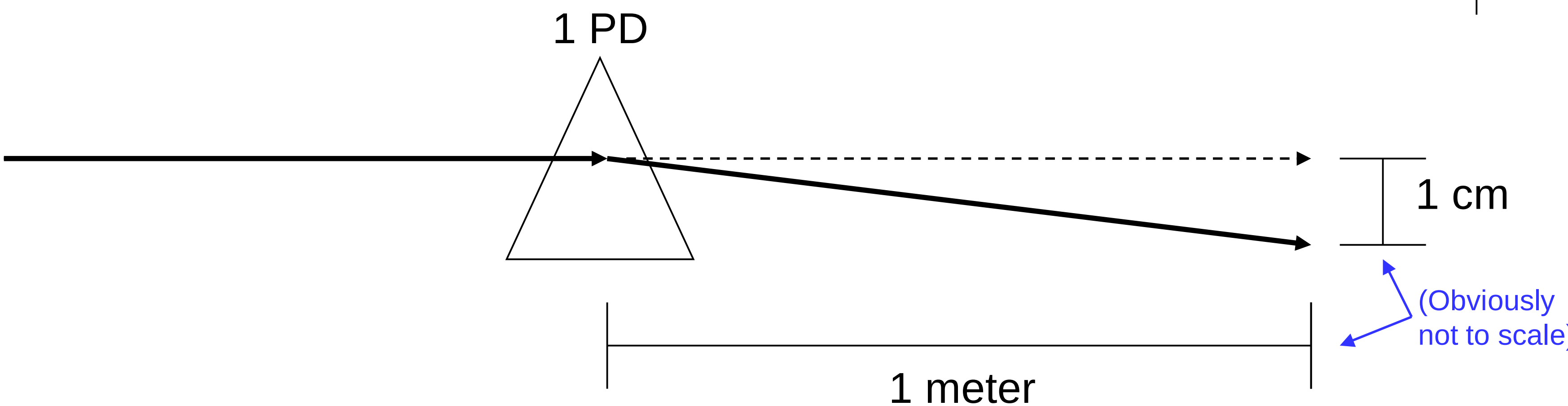


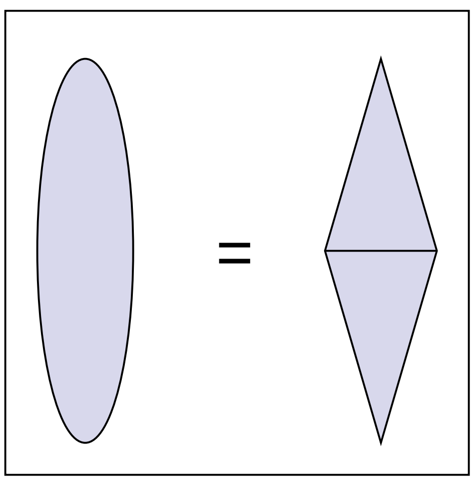
But, if we placed two prisms base-to-base or apex-to-apex, we could get light to converge and diverge, respectively



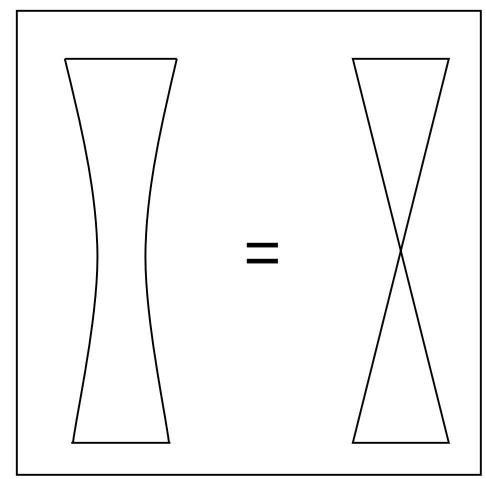
Vergence: T a



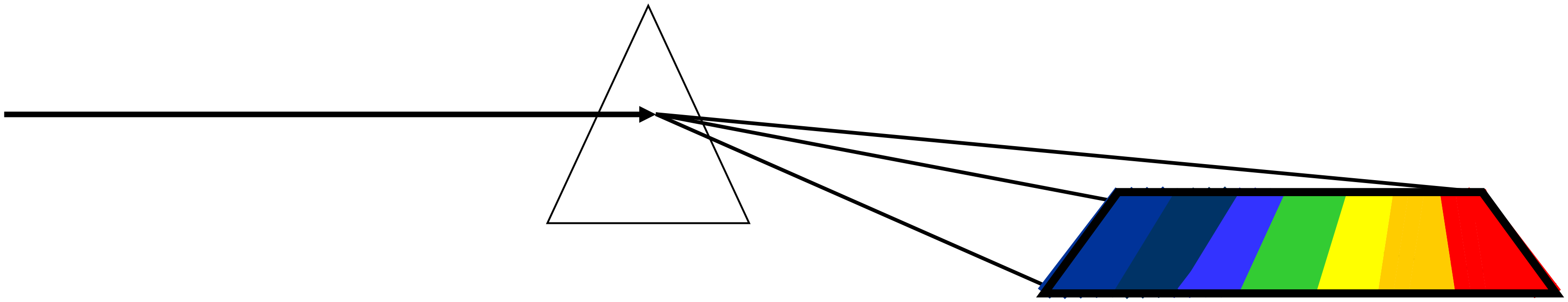




In fact, we will at times find it very useful to think of lenses as being composed of prisms arranged in just this manner!

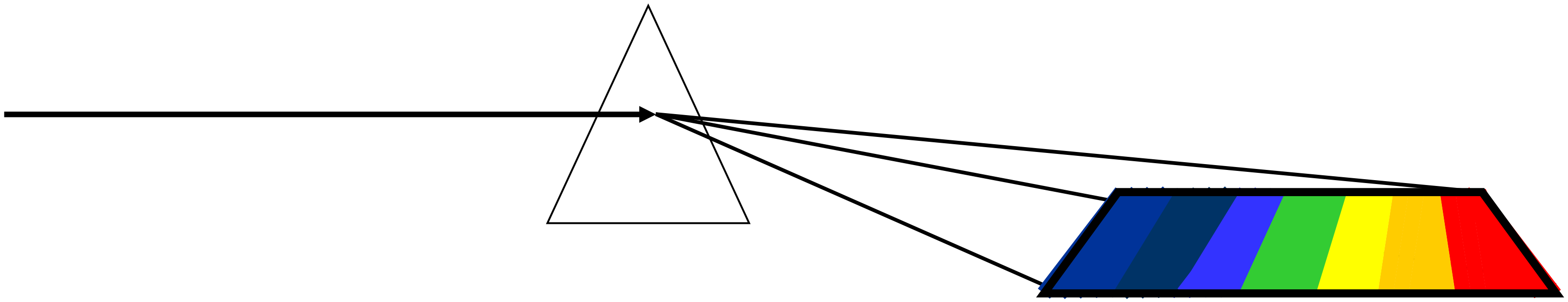


Vergence: The Vergence Formula

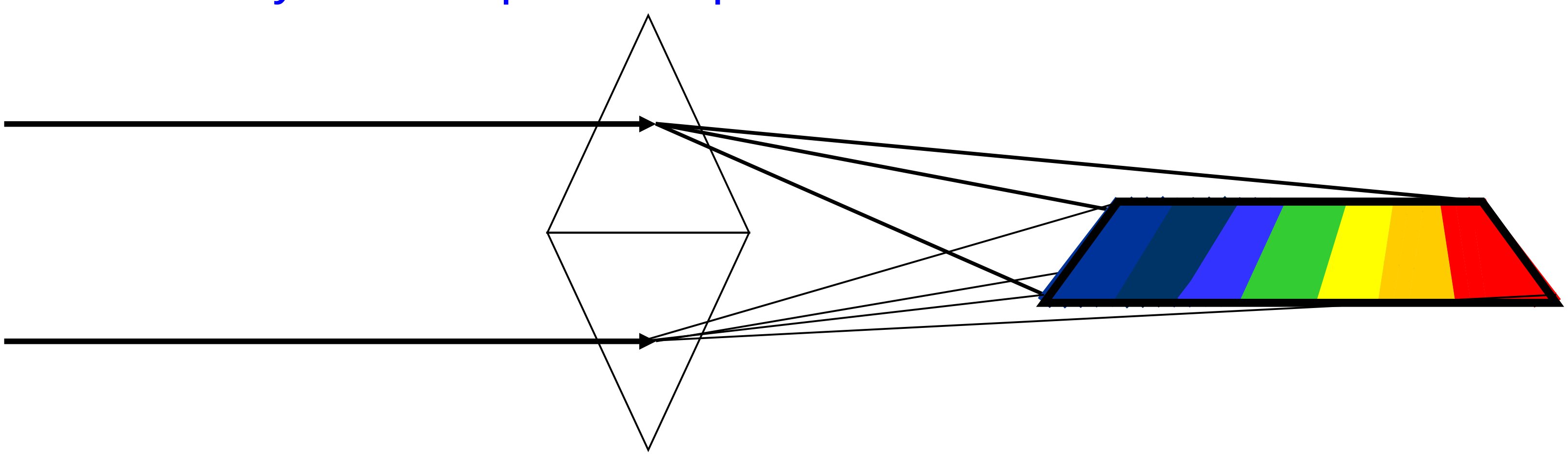


Of course, prisms also disperse white light into its component **colors.** They do this because the different wavelengths are refracted different amounts. And because they are composed of prisms…

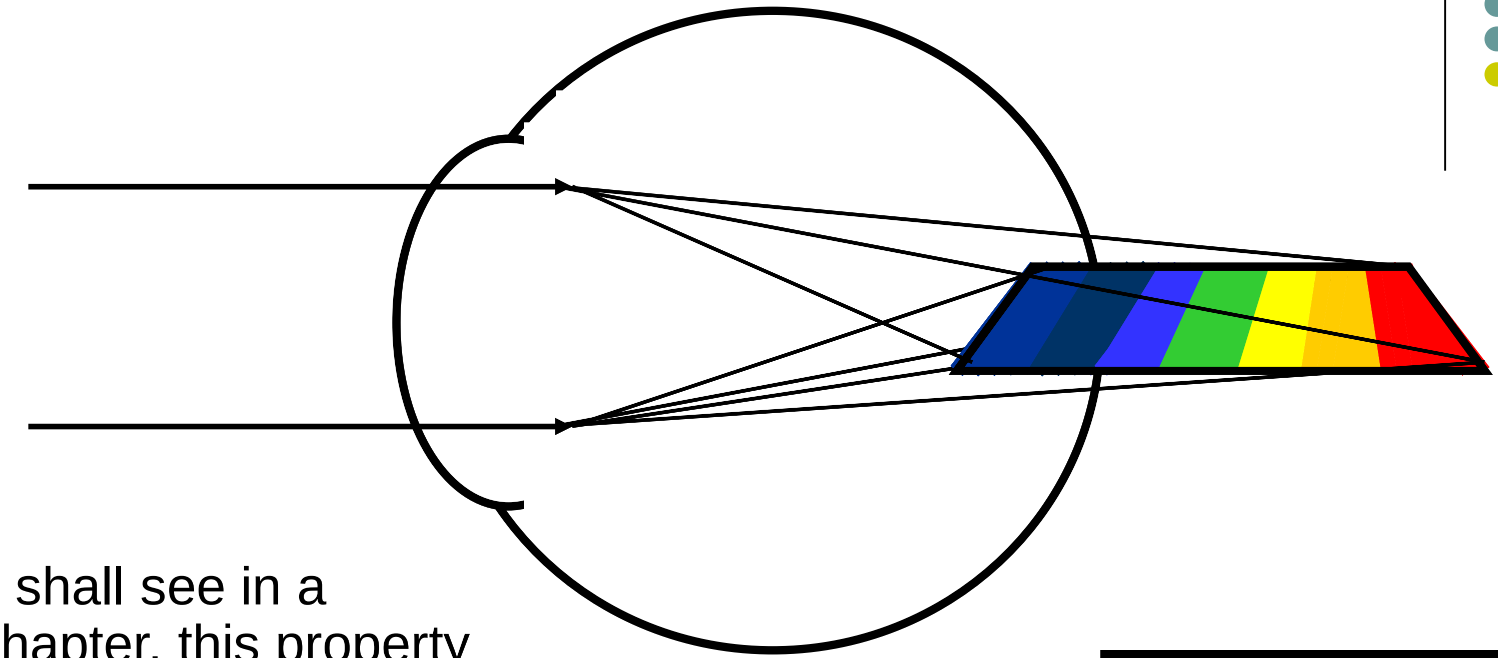
Vergence: The Vergence Formula



Of course, prisms also disperse white light into its component **colors.** They do this because the different wavelengths are refracted different amounts. And because they are composed of prisms…*lenses do too.*



Vergence: The Vergence Formula



As we shall see in a
later chapter, this property
accounts for an important ocular phenomenon called *chromatic aberration*.