## Astigmatic Refractive Error: Introduction

Basic Optics, Chapter 10

## Refractive Error and Its Correction: Review

1) The far point is the point in space conjugate to the retina when the eye is not accommodating
2) An eye's refractive status is a function of the location of its far point

Far Point

In Chapter 5, we learned that refractive error is fundamentally a Far Point problem. The far point of the myopic eye is just anterior to the cornea, whereas the far point of the hyperopic eye is behind the eye. Absent correction or accommodation, neither is in focus at distance.

## Refractive Error and Its Correction: Review



In Chapter 8, we employed the Error Lens concept to explain why the far point of the myopic and hyperopic eyes are located where they are.


## Refractive Error and Its Correction: Review



In Chapter 6, we summarized refraction thusly: Place a lens in front of an eye so that the secondary focal point of the lens coincides with the far point of the eye.


## Refractive Error and Its Correction: Review

Hyperopic Eye


Eye Error Lens: -2D
Corrective Lens Needed: +2

Myopic Eye


Corrective Lens Needed: -4

To offset the error lens, the corrective lens needs to be of equal but opposite power (except for the adjustment in power needed to account for vertex distance).

## Refractive Error and Its Correction: Review

Hyperopic Eye


Corrective Lens Needed: +2

Myopic Eye


Corrective Lens Needed: -4

To offset the error lens, the corrective lens needs to be of equal but opposite power (except for the adjustment in power needed to account for vertex distance).

But note that the discussion thus far has dealt solely with spherical refractive error (and therefore with spherical error lenses).

## Spherocylindrical Lenses



> By definition, a spherical lens has equal power in all meridia, and focuses parallel rays to a single point at its secondary focal point*

*Bear in mind we are discussing idealized lenses here. We will see in a chapter on Aberrations that a true point-focus is exceedingly hard to come by!

## Spherocylindrical Lenses

More plus power in this meridian
Less plus power in this meridian (you can tell because it's more (you can tell because it's less


In a spherocylindrical lens, however, the dioptric powers are not equal in all meridia, so light will not be focused to a single point!

## Spherocylindrical Lenses



In a spherocylindrical lens, however, the dioptric powers are not equal in all meridia, so light will not be focused to a single point!

## Spherocylindrical Lenses

## Spherocylindrical lens

Less plus power
in this meridian


How does a spherocylindrical lens focus light?

> To answer this, first consider that a spherocylindrical lens consists, in essence, of two cylindrical lenses of differing dioptric powers oriented $90^{\circ}$ apart

## Spherocylindrical Lenses



How does a spherocylindrical lens focus light?

> To answer this, first consider that a spherocylindrical lens consists, in essence, of two cylindrical lenses of differing dioptric powers oriented $90^{\circ}$ apart

## Spherocylindrical Lenses



How does a spherocylindrical lens focus light?

> To answer this, first consider that a spherocylindrical lens consists, in essence, of two cylindrical lenses of differing dioptric powers oriented $90^{\circ}$ apart

## Spherocylindrical Lenses

Spherical lens = Cylindrical lens $\quad$ Cylindrical lens


Equal power in all meridia

By the way....A spherical lens can be thought of as two cylindrical lenses of identical dioptric powers oriented $90^{\circ}$ apart...

## Spherocylindrical Lenses

Spherical lens $=$ Cylindrical lens + Cylindrical lens + Cylindrical lens


Equal power in all meridia

> ...or for that matter, as THREE cylindrical lenses of identical power oriented $60^{\circ}$ apart, or four at $45^{\circ}$, etc. But for now, just think of it as two identical cylinders at $90^{\circ}$ to one another.

## Spherocylindrical Lenses

Meridian of power
= 180

Note that a cylindrical lens has no power in the meridian of its axis!


Meridian of power

$$
=90
$$

Axis of power $=90$

Because a cylinder has power in only one meridian, there's no way it could focus parallel rays to a point.

## Spherocylindrical Lenses



> Axis of power $=90$

Because a cylinder has power in only one meridian, there's no way it could focus parallel rays to a point.

So then, how does a cylinder focus light?

## Spherocylindrical Lenses



## Spherocylindrical Lenses


(This is important! The separateness of the rays in the drawing seems to indicate that they originate at different locations on the source of origin. They do not! They originated from a single point, but are so far removed from that point that their relative vergence is now zero.)

Consider a cylinder, oriented as shown, that is encountering parallel rays from a point at infinity...

## Spherocylindrical Lenses

Because a cylindrical lens adds vergence in one meridian only...

Consider a cylinder, oriented as shown, that is encountering parallel rays from a point at infinity...

## Spherocylindrical Lenses

Because a cylindrical lens adds vergence in one meridian only...


Consider a cylinder, oriented as shown, that is encountering parallel rays from a point at infinity...

## Spherocylindrical Lenses



## Spherocylindrical Lenses



## Spherocylindrical Lenses

If we examined the light at various points...

## Spherocylindrical Lenses

If we examined the light at various points...

## Spherocylindrical Lenses

If we examined the light at various points...

## Spherocylindrical Lenses

If we examined the light at various points...

## Spherocylindrical Lenses

If we examined the light at various points...

## Spherocylindrical Lenses

If we examined the light at various points...

## Spherocylindrical Lenses

If we examined the light at various points... we would find it formed a series of rectangles that vary in width, but not height

## Spherocylindrical Lenses



With this cylinder orientation, the light forms a series of rectangles that vary in height, but not width

## Spherocylindrical Lenses



So, if a spherocylindrical lens is composed of two cylindrical lenses of different dioptric powers oriented $90^{\circ}$ apart...

## Spherocylindrical Lenses



So, if a spherocylindrical lens is composed of two cylindrical lenses of different dioptric powers oriented $90^{\circ}$ apart...a spherocylindrical lens must do something like this. Let's examine it in more detail.

## Spherocylindrical Lenses

Consider a spherocylindrical lens that is the dioptric equivalent of +1 D and +2 D cylinders oriented thusly:


## Spherocylindrical Lenses

Consider a spherocylindrical lens that is the dioptric equivalent of +1 D and +2 D cylinders oriented thusly:


Note: The $+1 D$ and $+2 D$ labels are pointing to the meridia of power (090 and 180, respectively). The axis of +1 power is at 180; of $+2 D, 090$.


## Spherocylindrical Lenses

Consider a spherocylindrical lens that is the dioptric equivalent of +1 D and +2 D cylinders oriented thusly:


Note: The $+1 D$ and $+2 D$ labels are pointing to the meridia of power (090 and 180, respectively). The axis of $+1 D$ power is at 180; of $+2 D, 090$.

Let's examine the light as it moves along its post-refraction path

## Spherocylindrical Lenses



What the light would look like at various locations along its post-refraction path

The rays are converging in both vertical and horizontal aspects. The horizontal is converging faster because there is more power in that meridian.

## Spherocylindrical Lenses



What the light would look like at various locations along its post-refraction path

## Spherocylindrical Lenses



What the light would look like
The rays have converged further to form a vertical focal line. Note that the rays are continuing to converge vertically as well (i.e., the line is shorter than the previous oval). What is the distance from the lens to this anterior focal line?

## Spherocylindrical Lenses



What the light would look like
The rays have converged further to form a vertical focal line. Note that the rays are continuing to converge vertically as well (i.e., the line is shorter than the previous oval). What is the distance from the lens to this anterior focal line?
A +2D cylinder will form a focal line at a distance of $1 / 2=.5$ meters.

## Spherocylindrical Lenses



The rays are continuing to converge vertically, so the height of the figure continues to shrink. However, the horizontal rays, having converged to form a focal line, are now diverging. Thus the width is now increasing.

## Spherocylindrical Lenses



The rays continue to get shorter and wider until...

## Spherocylindrical Lenses



## Spherocylindrical Lenses



## Spherocylindrical Lenses



## Spherocylindrical Lenses



## Spherocylindrical Lenses



Let's get back to the area we skipped. As it passes through this location, the pattern of light is morphing from a vertical oval to a horizontal oval. Does it make sense that, at some point in this process, the light pattern will form...

## Spherocylindrical Lenses



## Spherocylindrical Lenses



## Spherocylindrical Lenses



## Spherocylindrical Lenses



What is the distance from the lens to the circle?
To answer this we first need to determine.
What is the dioptric power associated with the circle?

## Spherocylindrical Lenses


(Spherical equivalent = average power of the two cylinder lenses. In other words, a spherocylindrical lens composed of a +1D and a +2D meridian focuses light, on average, like a +1.50 spherical lens.)

What is the distance from the lens to the circle?
To answer this we first need to determine.
What is the dioptric power associated with the circle? The dioptric power is the spherical equivalent (S.E.) of the lens--in this case, +1.50D.

## Spherocylindrical Lenses



What is the distance from the lens to the circle?
To answer this we first need to determine..
What is the dioptric power associated with the circle?
The dioptric power is the spherical equivalent (S.E.)
of the lens--in this case, +1.50D. Therefore, the distance is...

## Spherocylindrical Lenses



Note that this circle (about which we will have much more to say shortly) is located at the dioptric 'halfway point' between the focal lines (i.e., 1.5 is halfway between 1.0 and 2.0). But be sure to note also that the dioptric halfway point is not the same as the geometric halfway point (which would be at .75 m in this case).

## Spherocylindrical Lenses



So at last, we can see now how a spherocylindrical lens focuses parallel rays-not to a single secondary focal point, but rather to a pair of secondary focal lines separated by a circle of as yet unidentified significance. (To be continued in the next chapter.)

