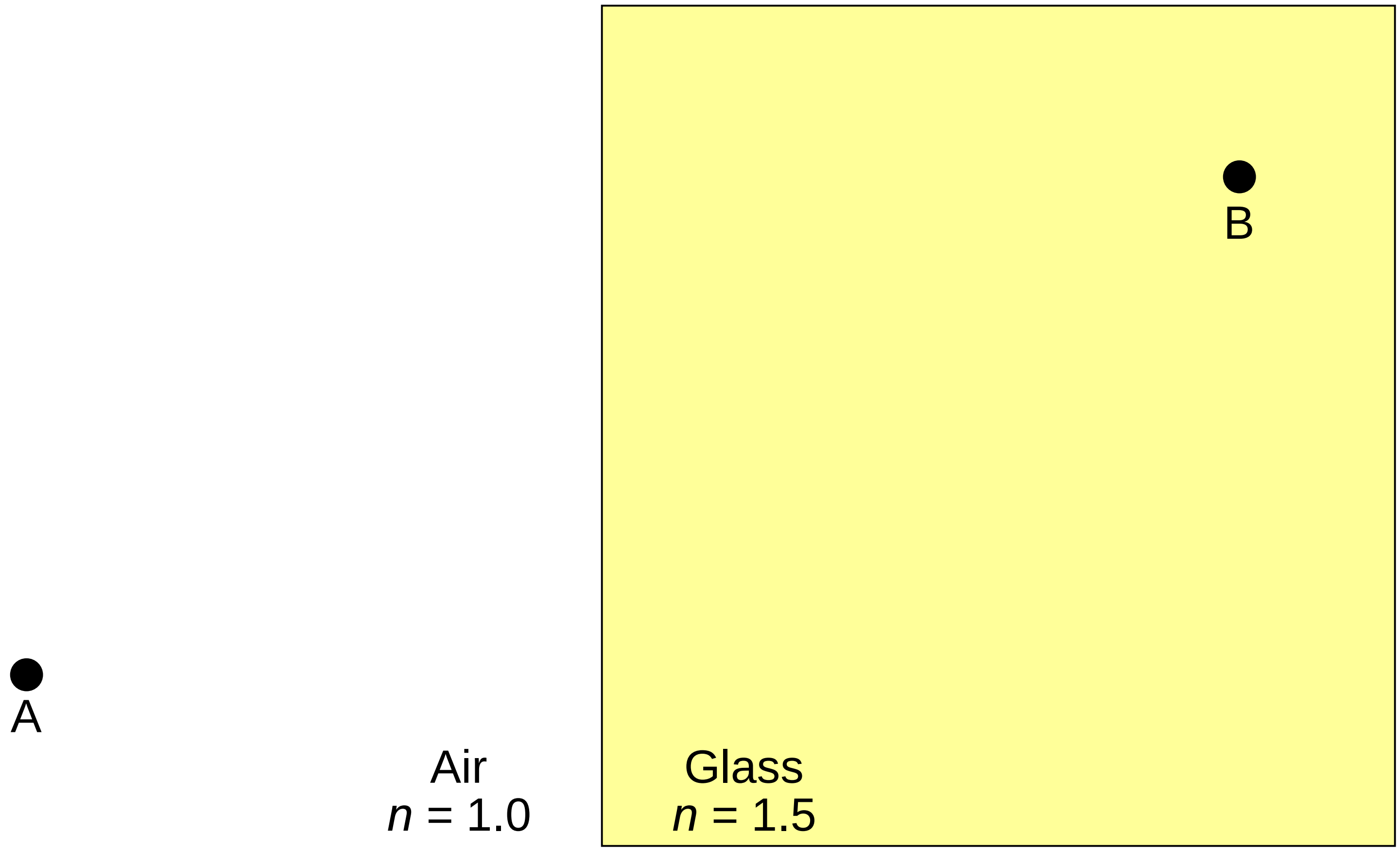
Refraction: Snell's Law

Basic Optics, Chapter 17

Refraction

Let’s talk about the extent to which light will be refracted at a given interface. To get us started, consider a simple thought experiment: What is the most ***efficient*** way for light to get from point *A* to point *B*?



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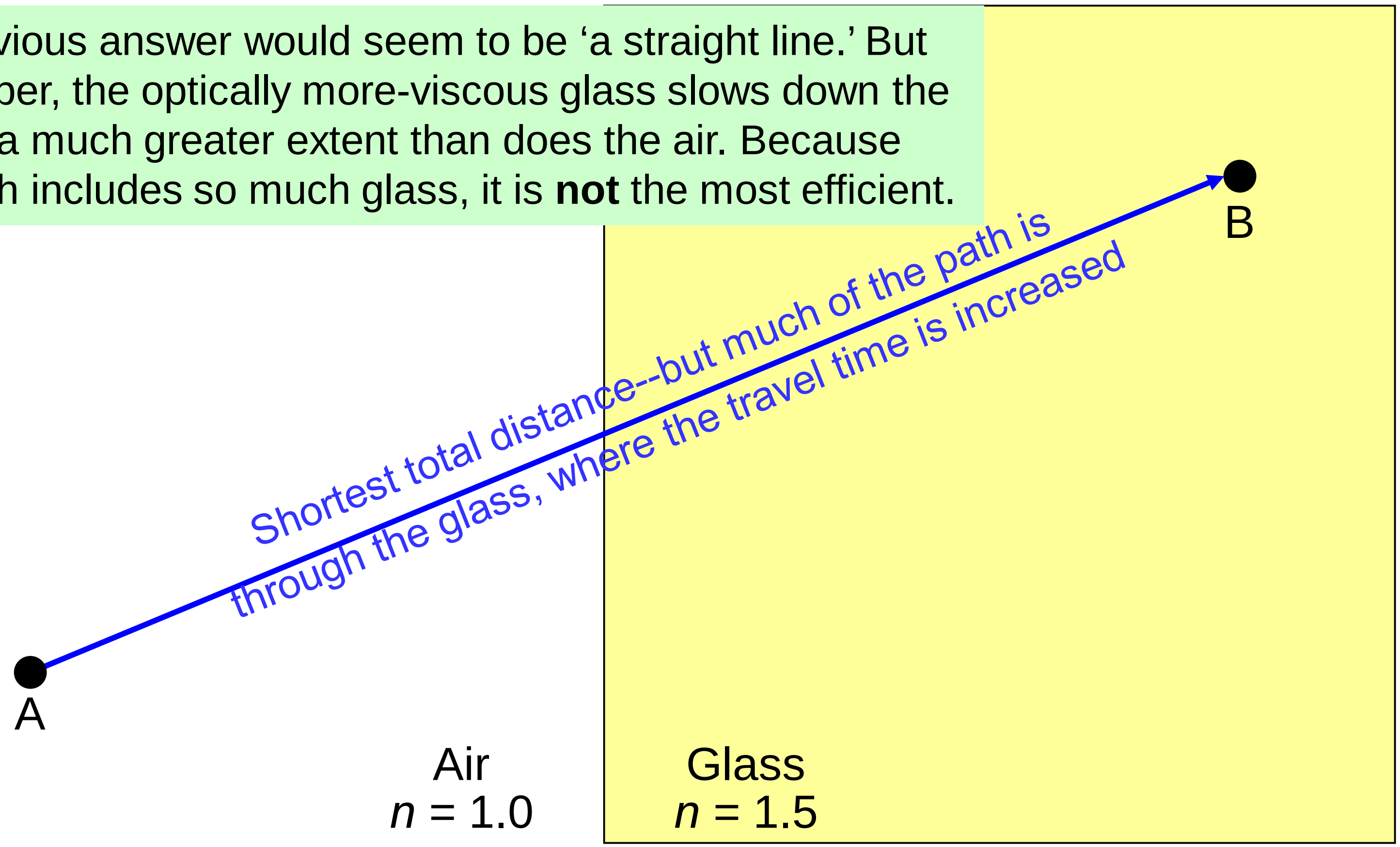
The obvious answer would seem to be 'a straight line.' But remember, the optically more-viscous glass slows down the light to a much greater extent than does the air. Because this path includes so much glass, it is **not** the most efficient.



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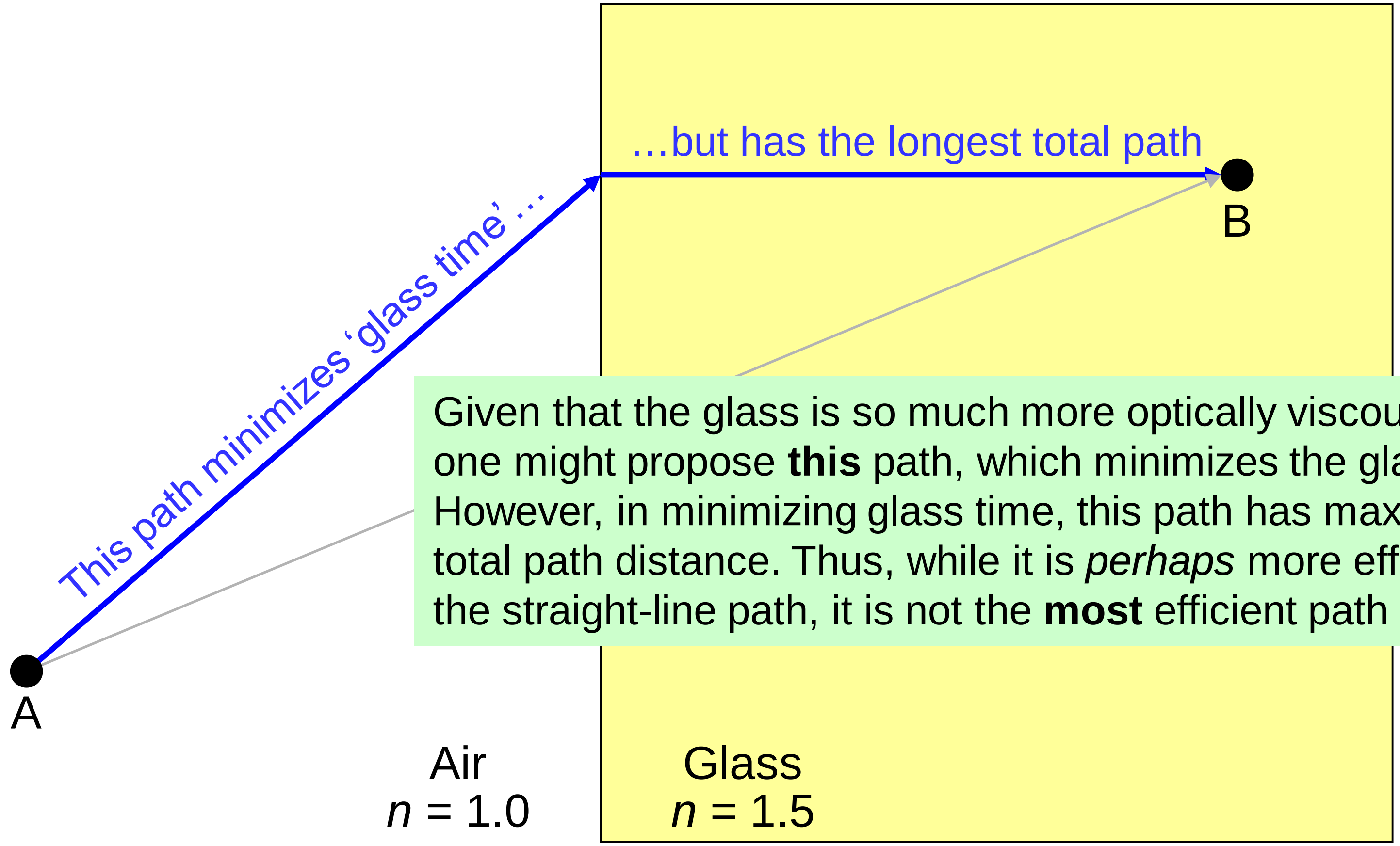

Given that the glass is so much more optically viscous than air, one might propose **this** path, which minimizes the glass portion. However, in minimizing glass time, this path has maximized total path distance. Thus, while it is *perhaps* more efficient than the straight-line path, it is not the **most** efficient path from A to B.

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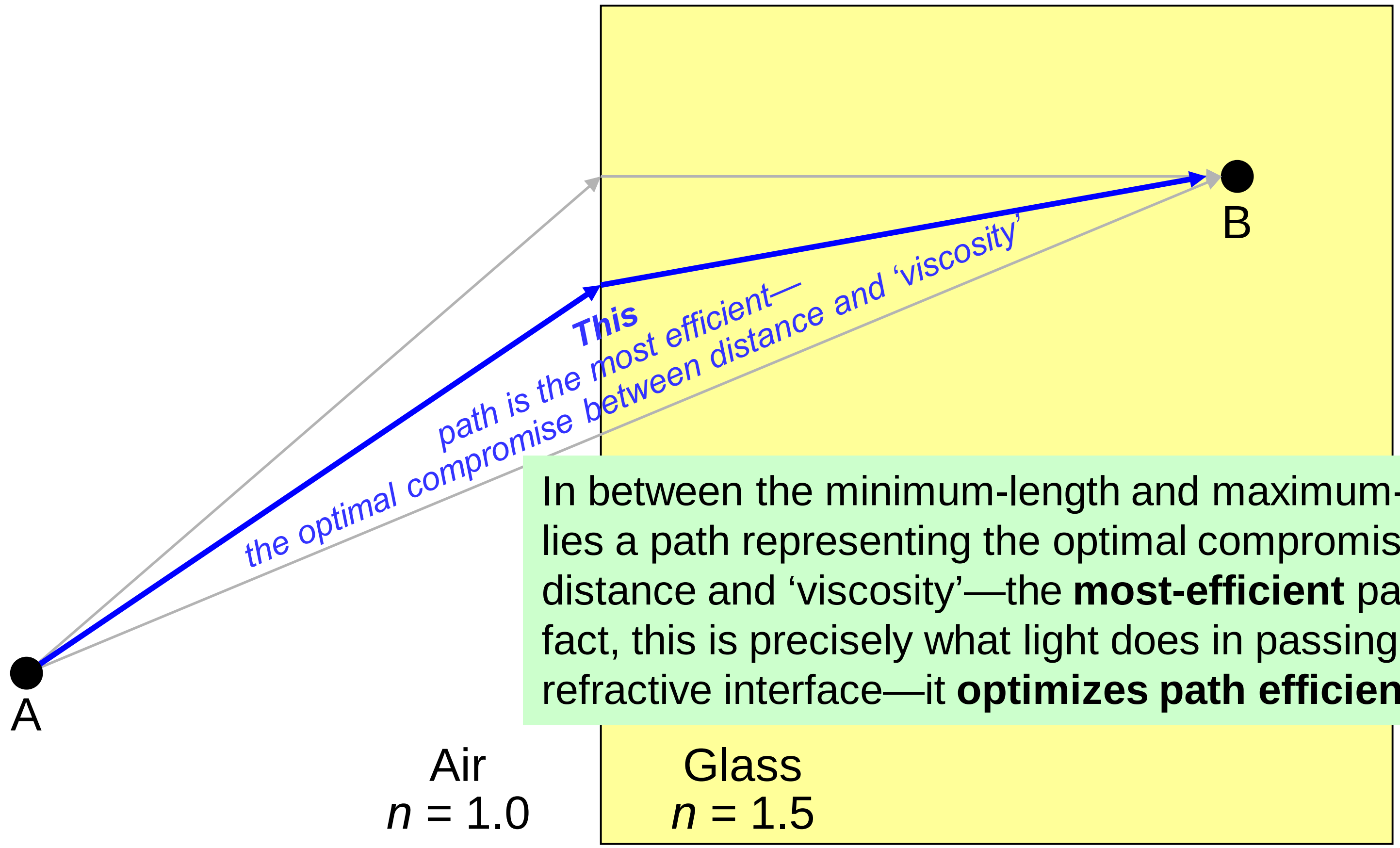

In between the minimum-length and maximum-length paths lies a path representing the optimal compromise between distance and 'viscosity'—the **most-efficient** path. And in fact, this is precisely what light does in passing through a refractive interface—it **optimizes path efficiency**.

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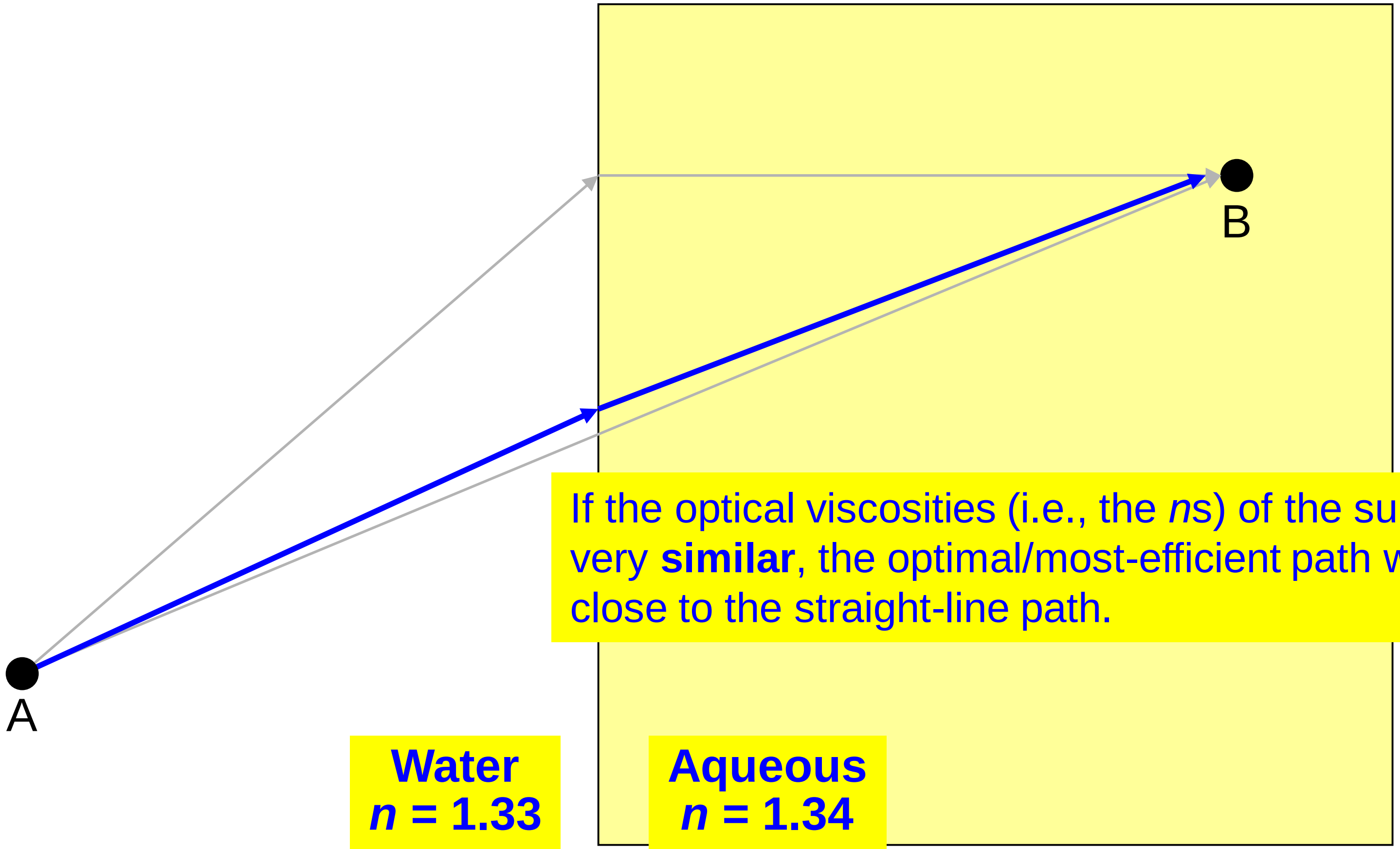

If the optical viscosities (i.e., the *n*s) of the substances are very **similar**, the optimal/most-efficient path will be very close to the straight-line path.

Water
***n* = 1.33**

Aqueous
***n* = 1.34**

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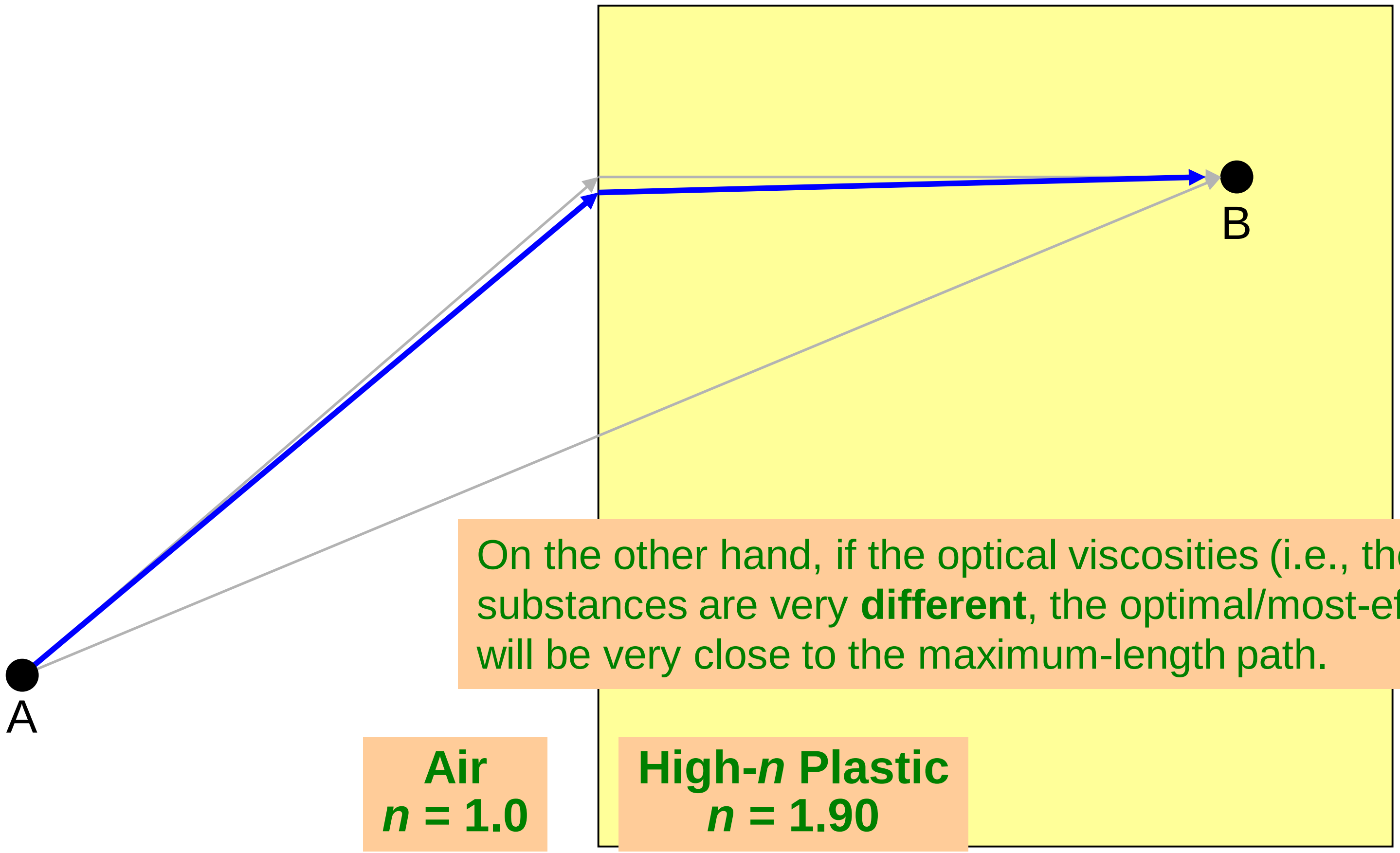

On the other hand, if the optical viscosities (i.e., the *n*s) of the substances are very **different**, the optimal/most-efficient path will be very close to the maximum-length path.

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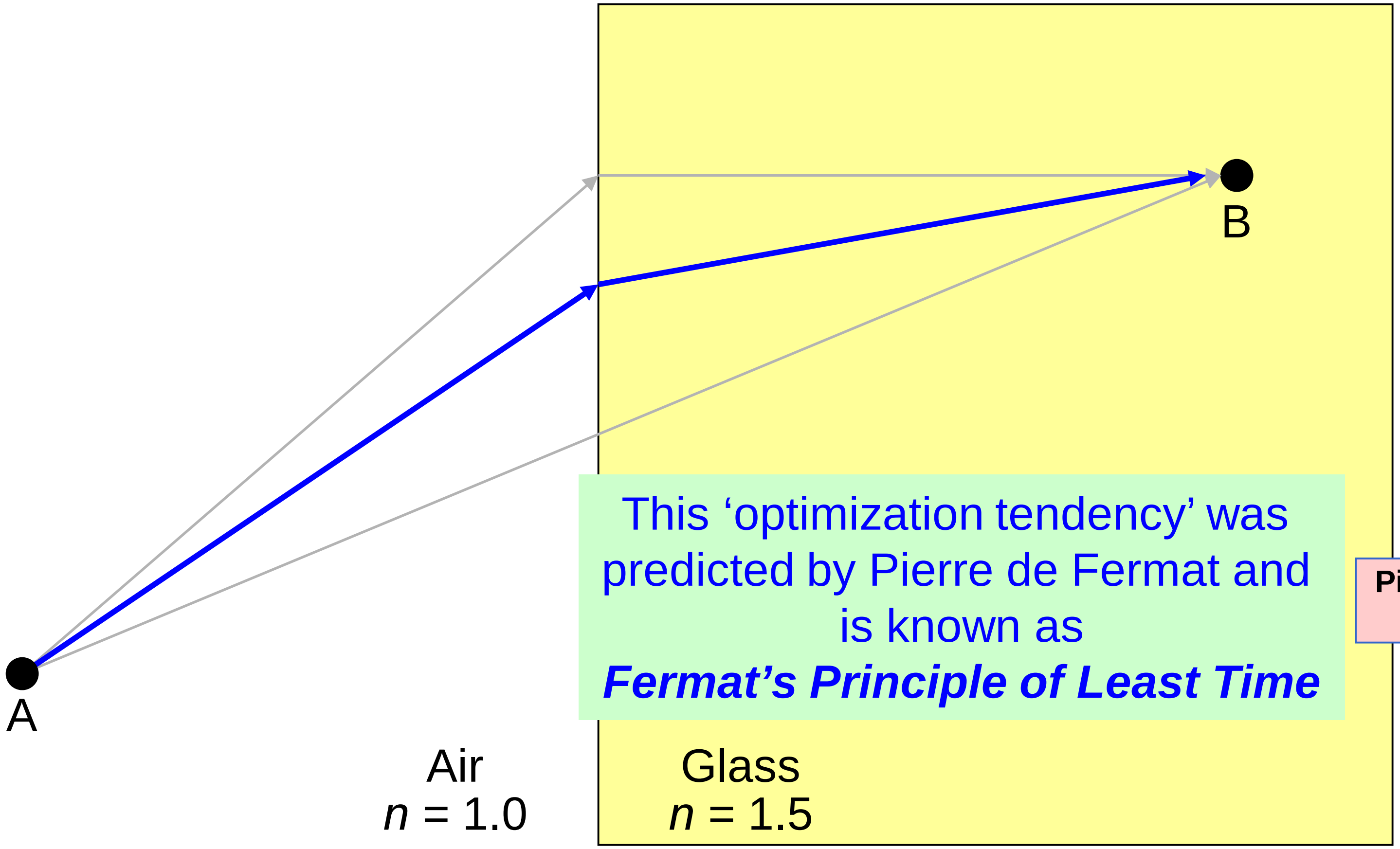
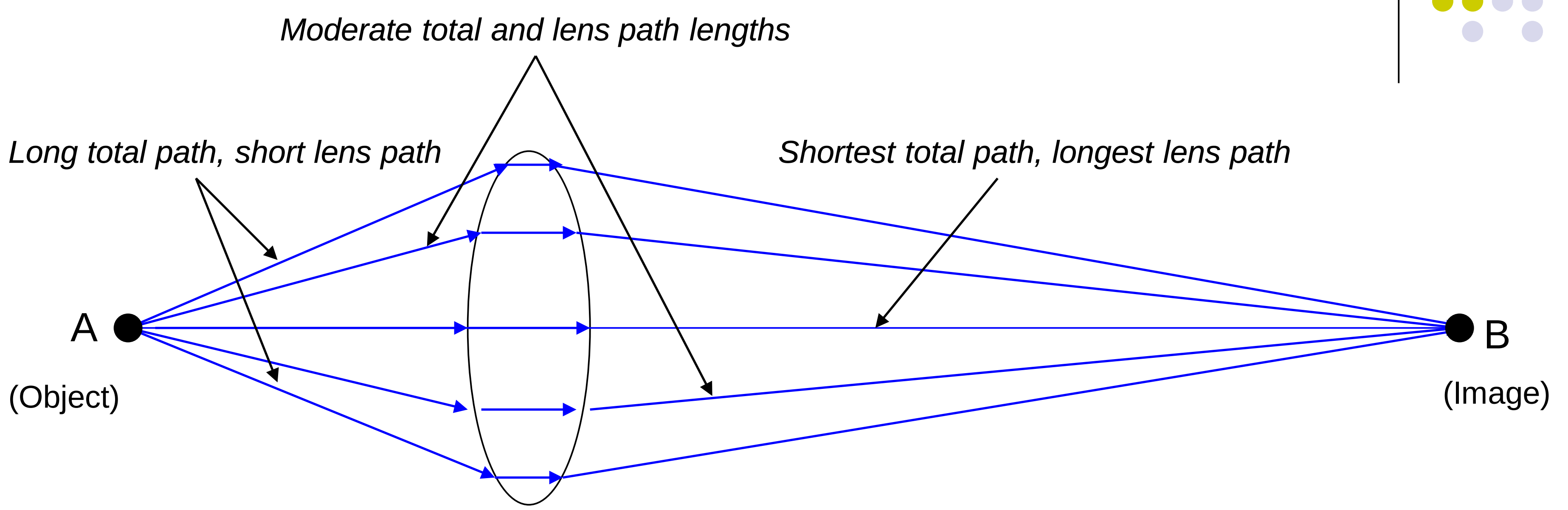


This ‘optimization tendency’ was predicted by Pierre de Fermat and is known as
Fermat’s Principle of Least Time

Pierre de Fermat
1601 - 1665

Refraction



Fermat's principle encapsulates the challenge facing lensmakers: *Fashion a lens such that every possible pathway from point A to point B has the same travel time.* If this is done, perfect focus will be achieved!

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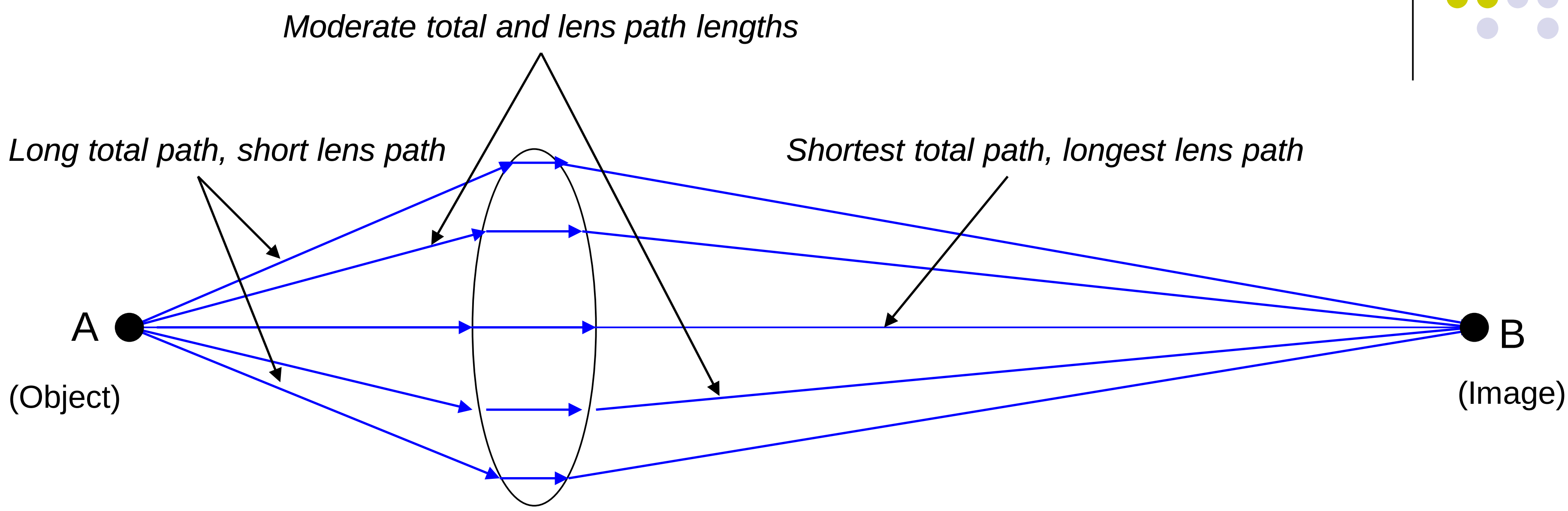

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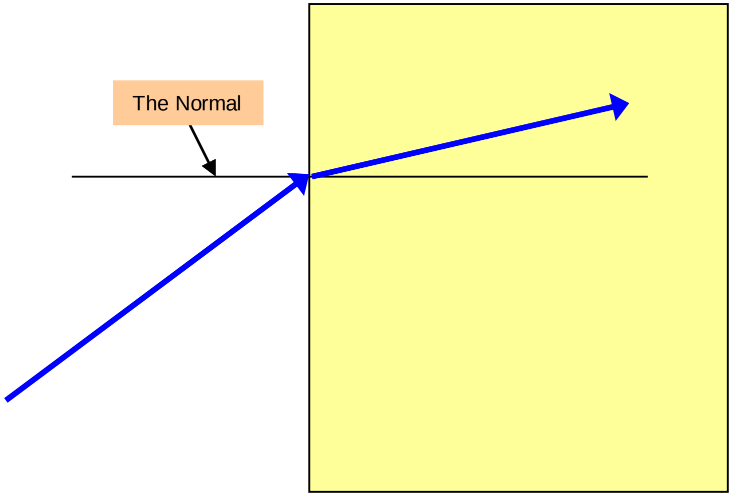
But Fermat's principle provides only a *qualitative* description of the behavior of light at a refractive interface. Precise lensmaking requires a **quantitative** description of refraction--as does scoring well on the OKAPs. Now that we have an intuitive feel for refraction, let's delve into its **quantification**.

Refraction

Willebrord (yes, Willebrord!) **Snell**
1580 - 1626

The 'optimization tendency' is quantified in the law of refraction: ***Snell's law***

$$n_i \sin \theta_i = n_t \sin \theta_t$$



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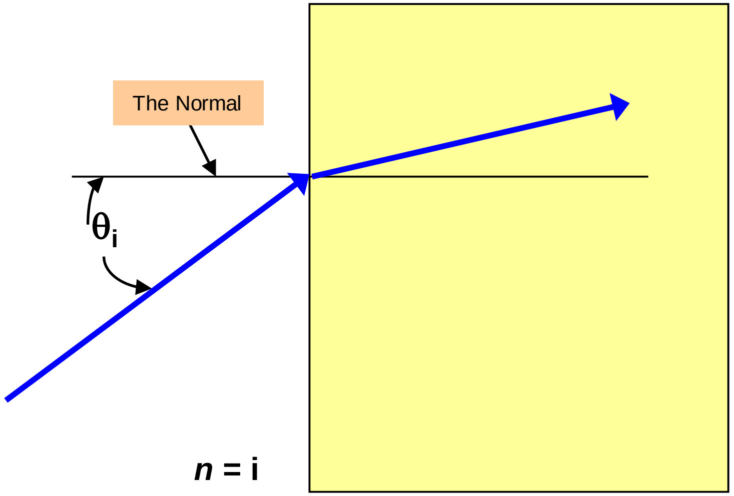
Angle of incidence with respect to the Normal



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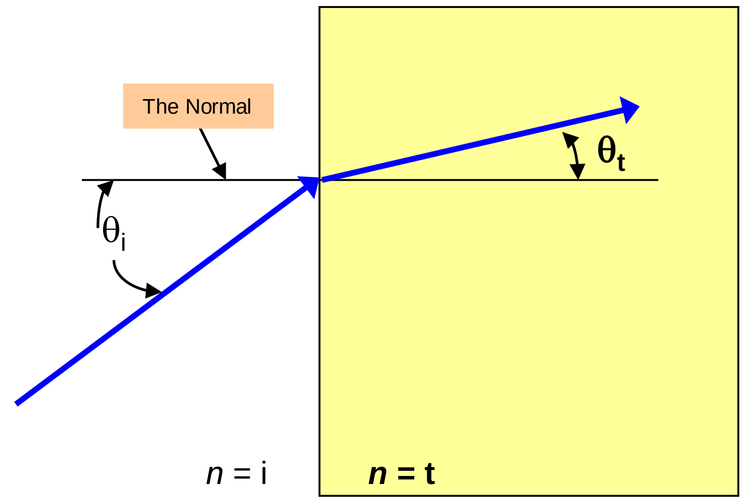

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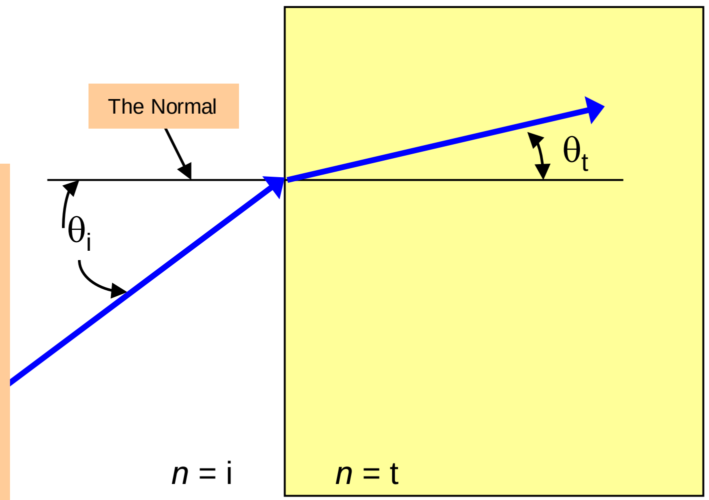
Snell's law may seem inelegant in comparison to the intuitive simplicity of Fermat's principle. However, Snell's law makes several non-obvious predictions concerning the behavior of light as it passes from a medium of higher *n* to one of lower *n*—predictions that have proved both accurate and extremely useful…



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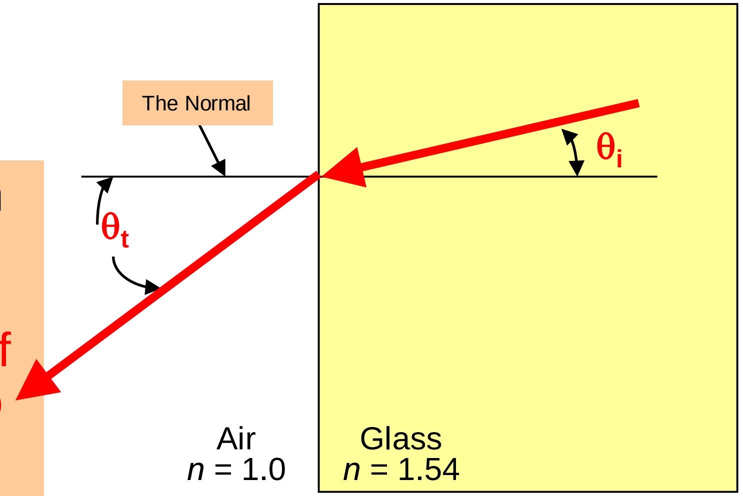

Note that the direction of light above has been reversed and the angles renamed!

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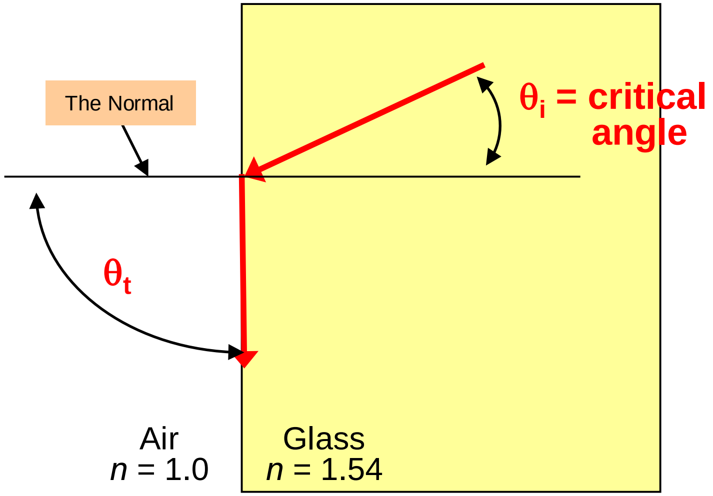

At a specific angle of incidence—called the ***critical angle***—the light will be refracted 90° to the normal, i.e., it will skid along the interface without passing through it. And…

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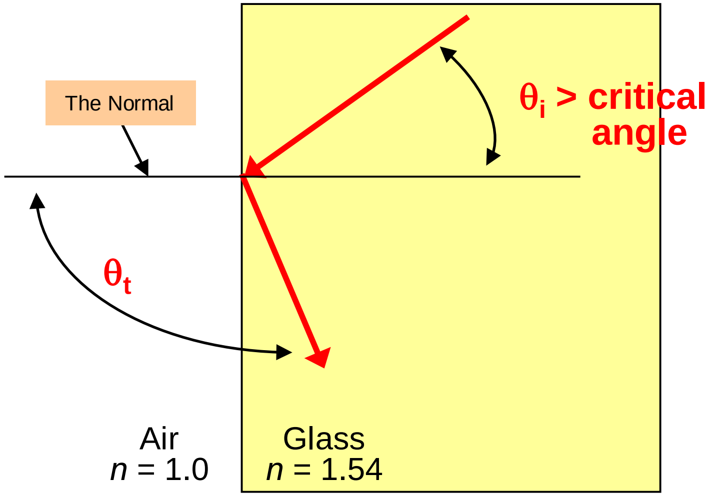

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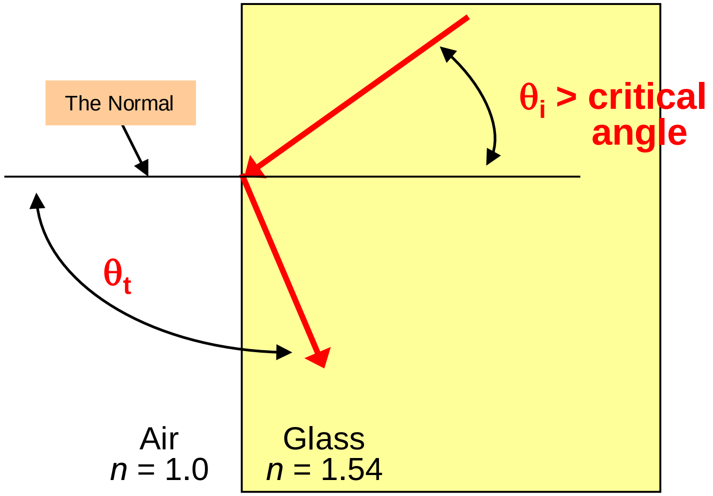

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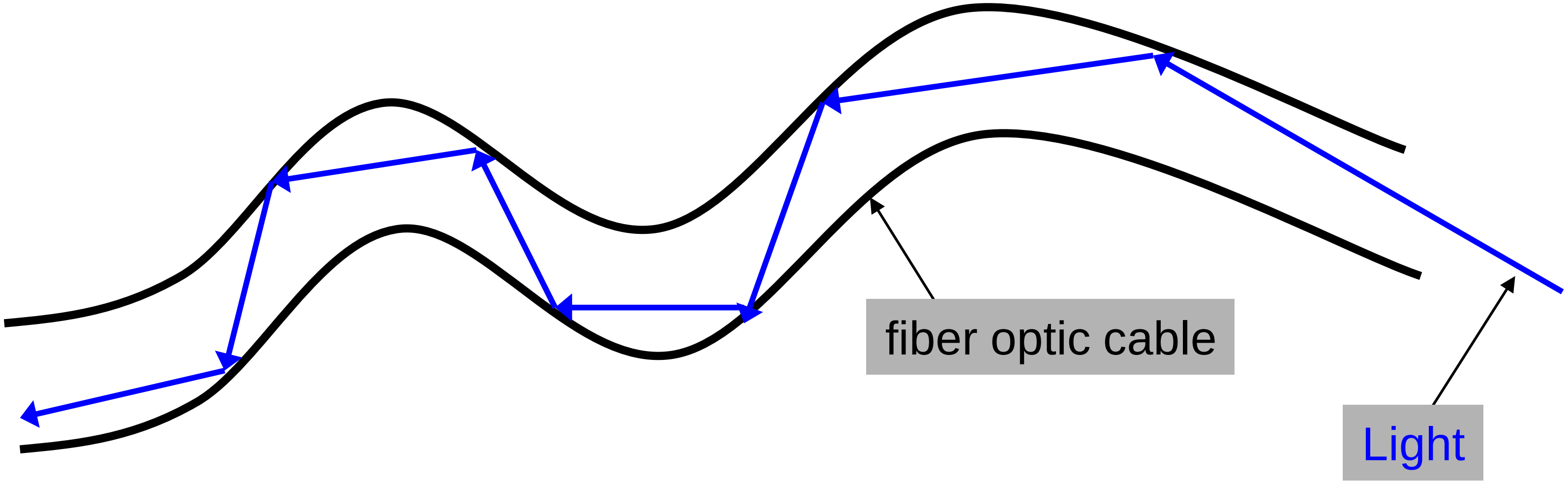
This phenomenon—*total internal reflection*—allows *fiber optic communications*.

Refraction

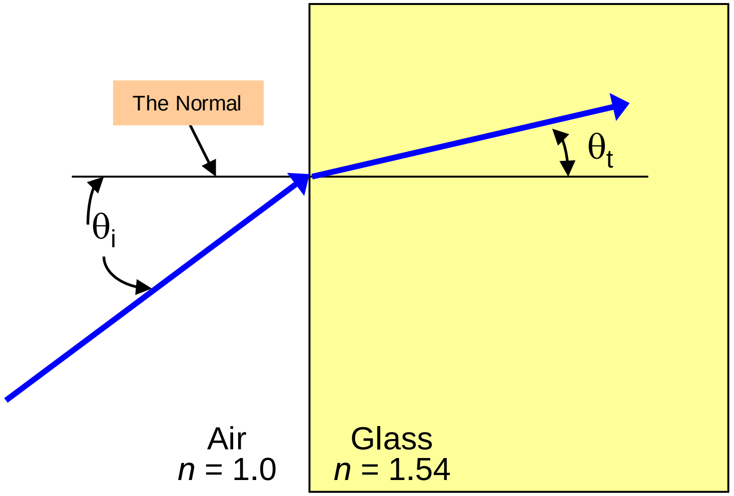
Total internal reflection



Using total internal reflection, a tremendous amount of information can be transmitted great distances with very little loss of fidelity.

Refraction

Thus far we've discussed refraction at a flat surface…



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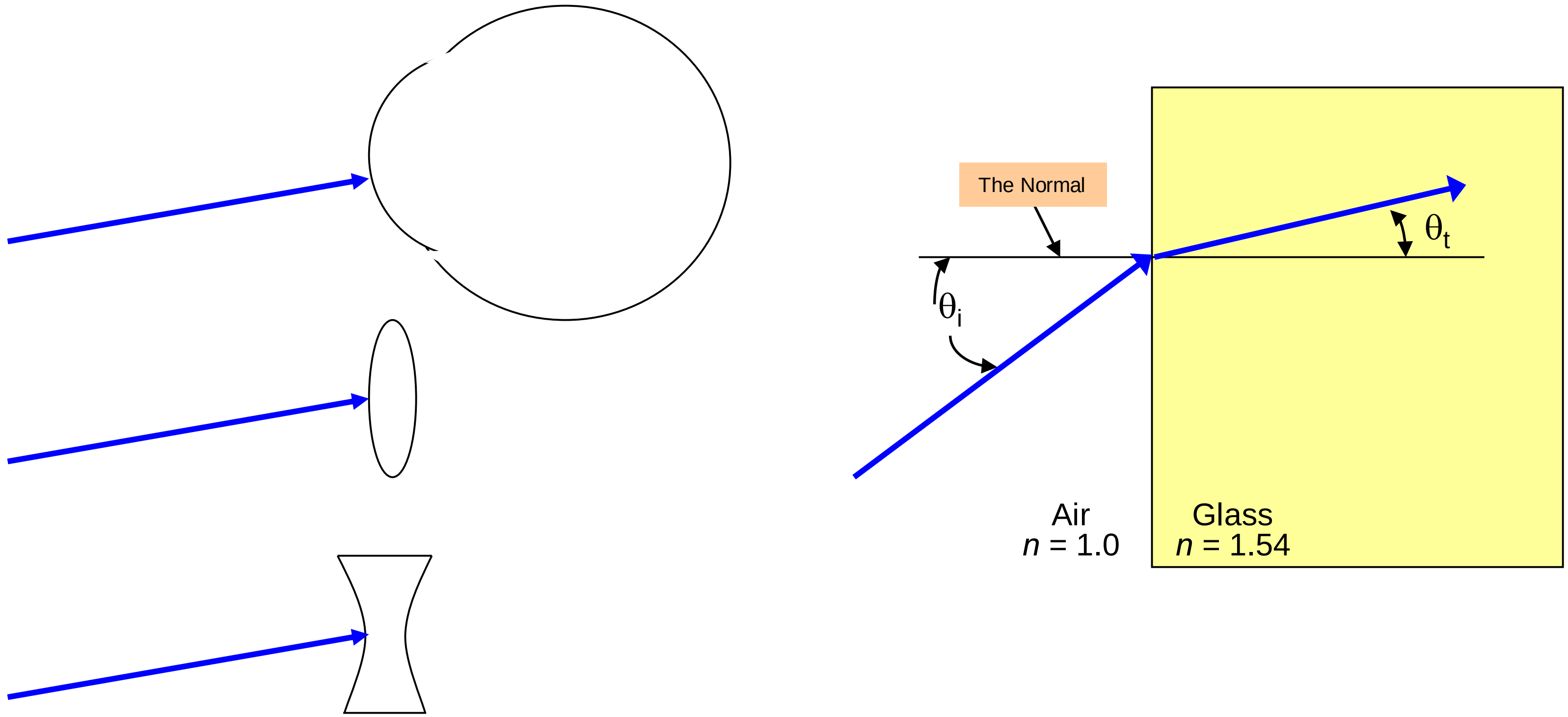

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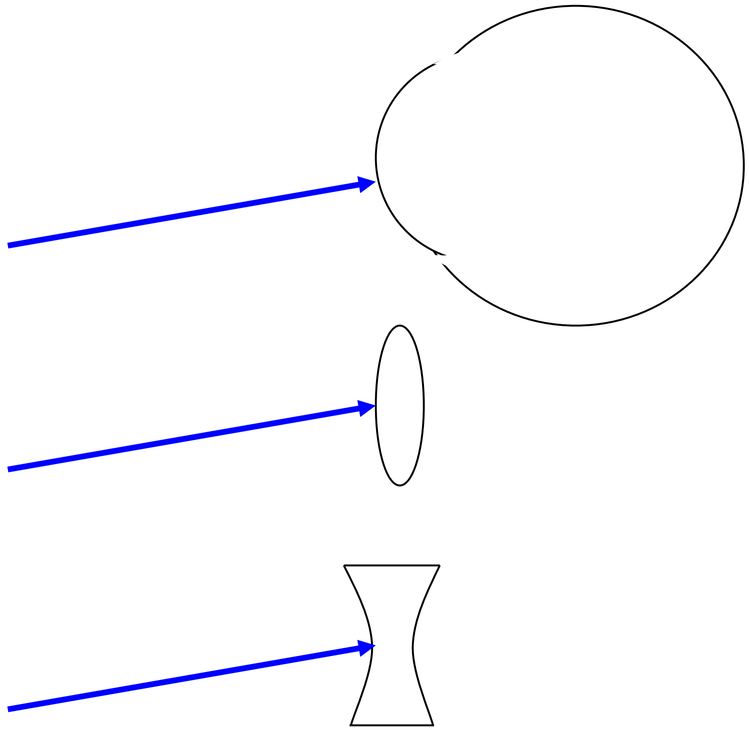

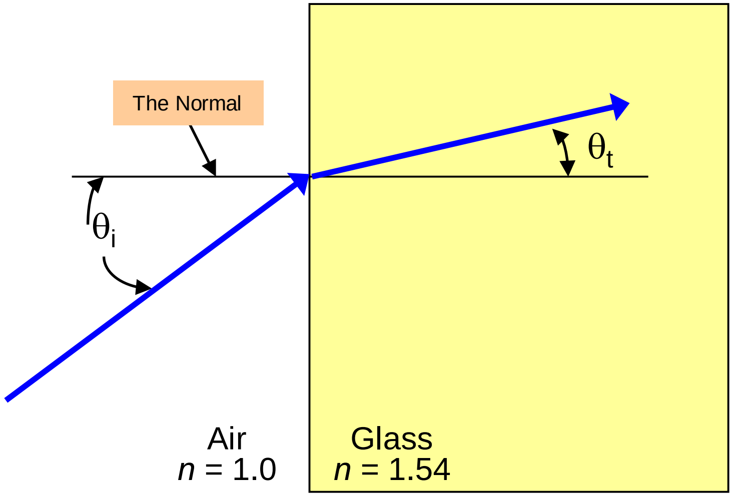

Snell's law still rules: light rays will be transmitted as a function of the normal, the relative *n*s of the materials, and the angle of incidence. But this begs the question:

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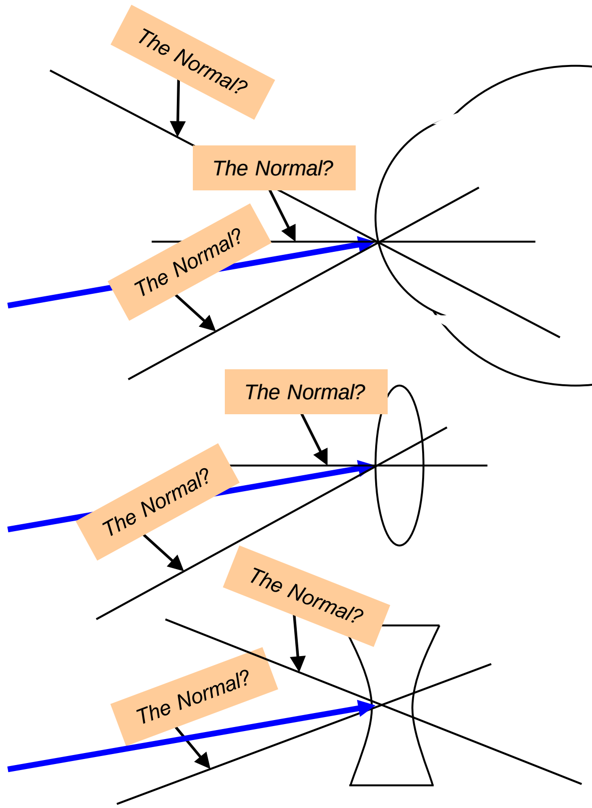
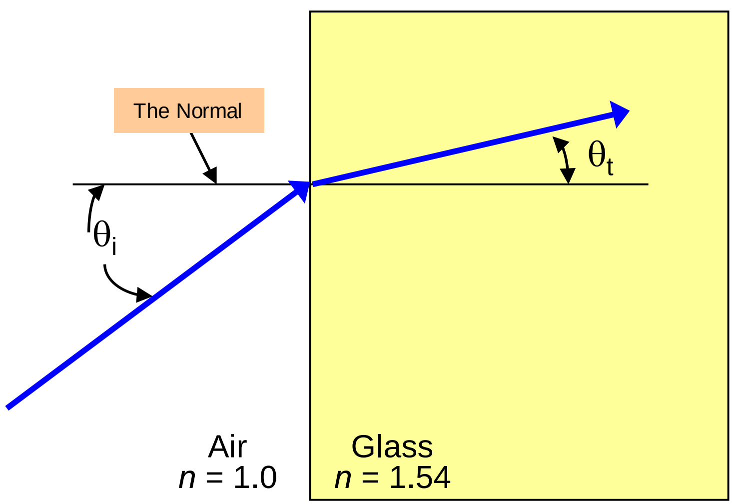
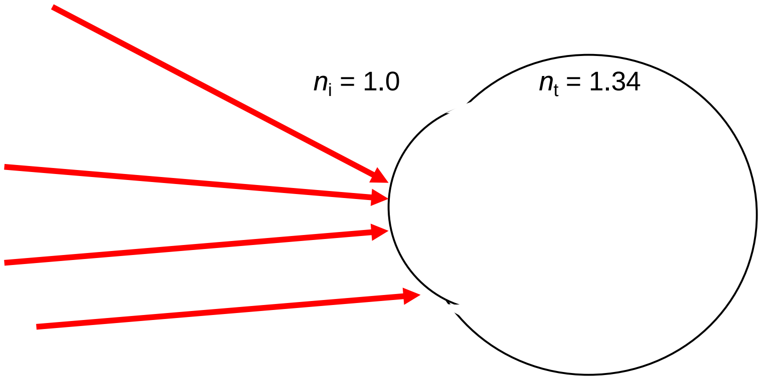




Snell's law still rules: light rays will be transmitted as a function of the normal, the relative *n*s of the materials, and the angle of incidence. But this begs the question: ***Where's the normal?***

Refraction



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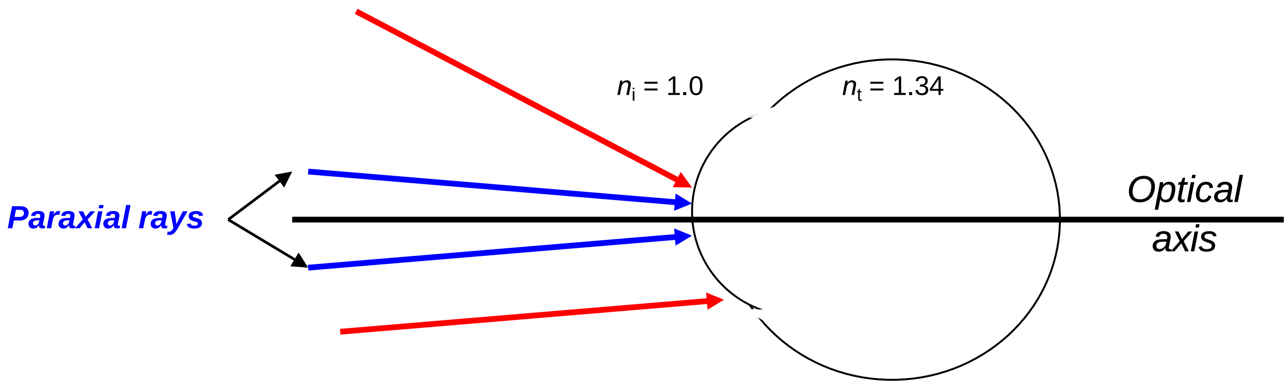

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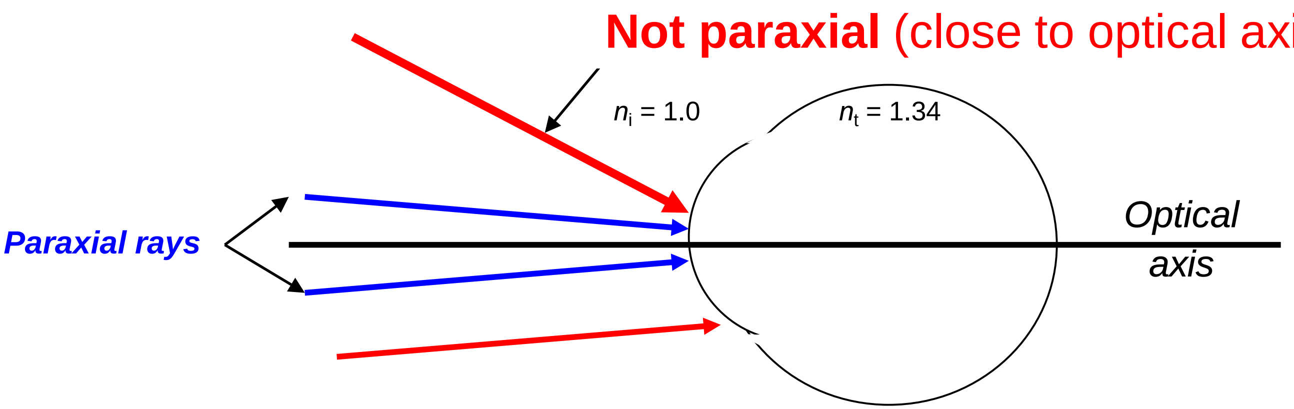
When dealing with refraction at a curved surface, we work only with the ***paraxial rays:*** *Those that are both* ***close to the*** ***optical axis*** *and* ***nearly parallel to it.***

(We will define the term *optical axis* in Chapter 18. Suffice to say for the moment that it is ***not*** the same thing as the normal.)

Refraction



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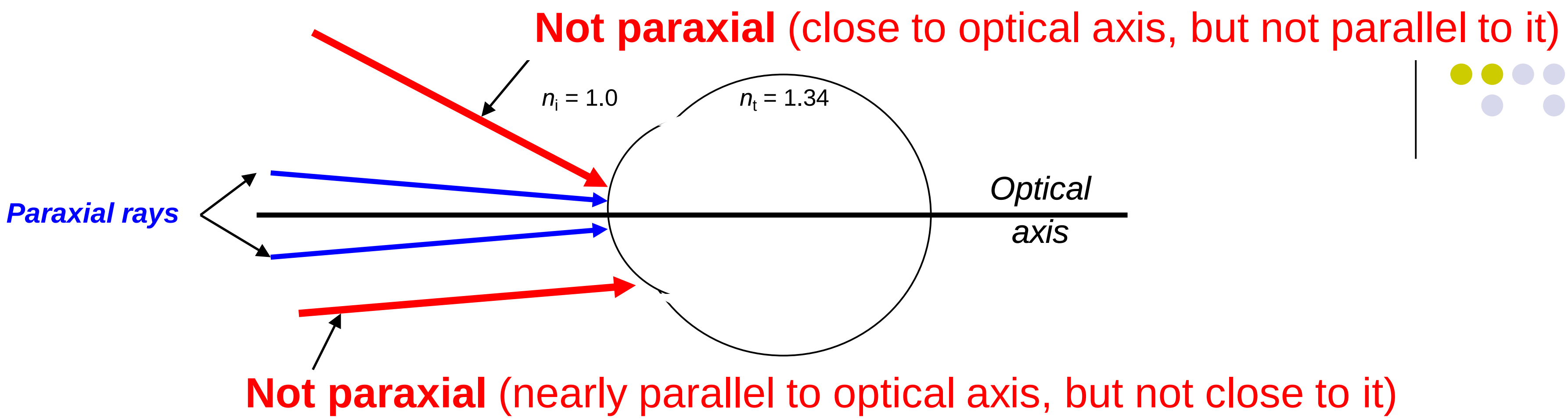

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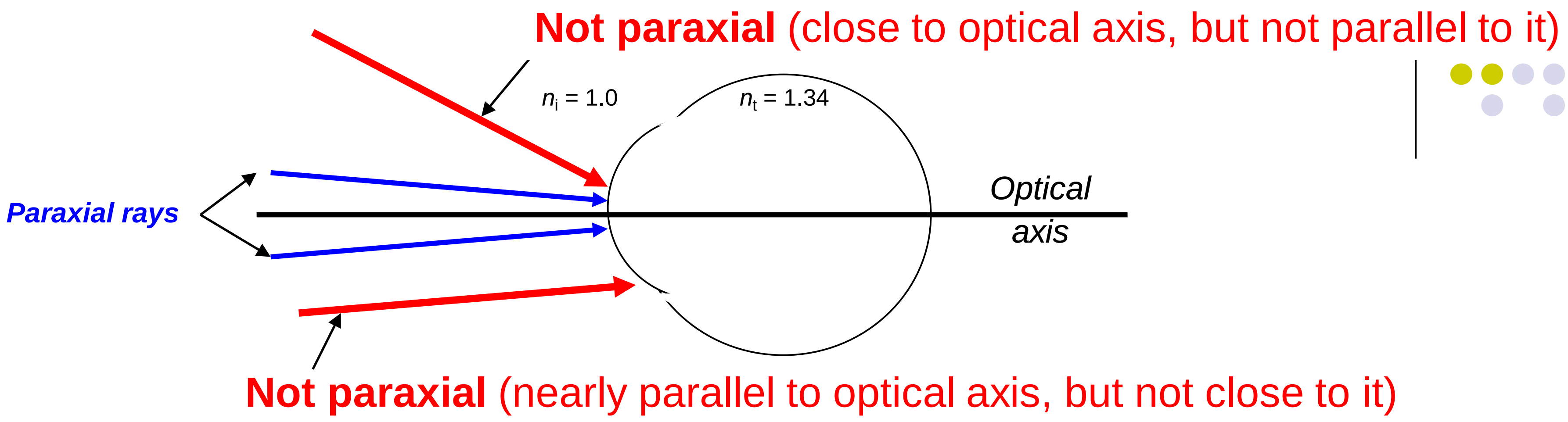

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This so-called *paraxial assumption* is extremely important, because it allows us to treat all of the relevant light rays as an aggregate, rather than individually. That is…

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$$n_i \sin \theta_i = n_t \sin \theta_t$$

Power (in diopters, *D*) =

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n *where the rays are* **going**

n *where the rays are* **coming from**

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Radius of curvature *of the interface*

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Note:

1) The power goes up as the difference in *n* **increases**, or as the radius **decreases**
2) Given that *radius of curvature* is in the power formula, it follows that the refracting interface must be **spherical** in shape (only spherical surfaces have a single radius of curvature).

An important but oft-ignored assumption!