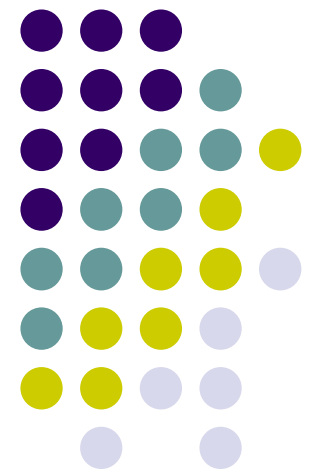


# Astigmatic Refractive Error: The Conoid of Sturm

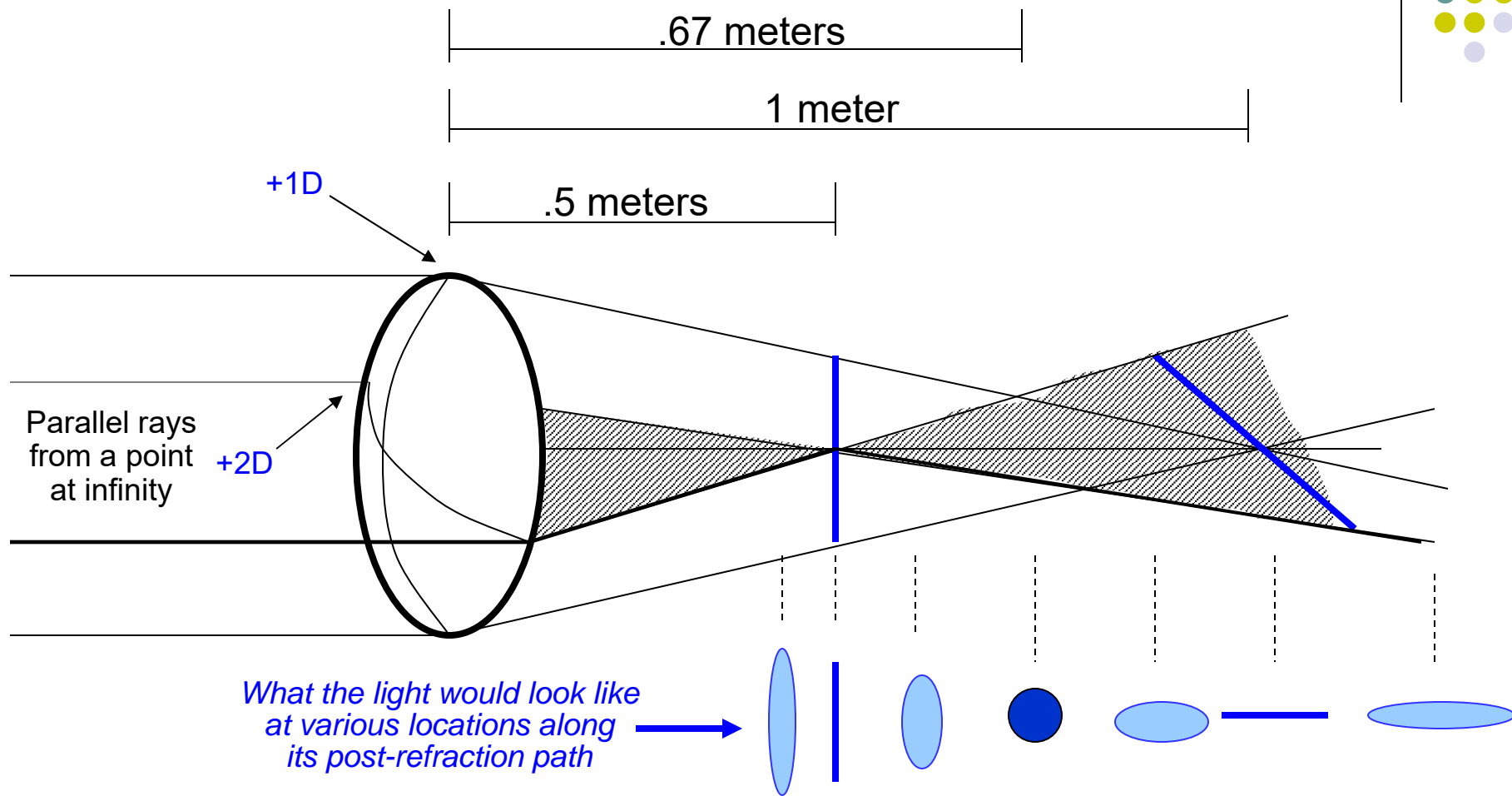
---

*Basic Optics*, Chapter 11





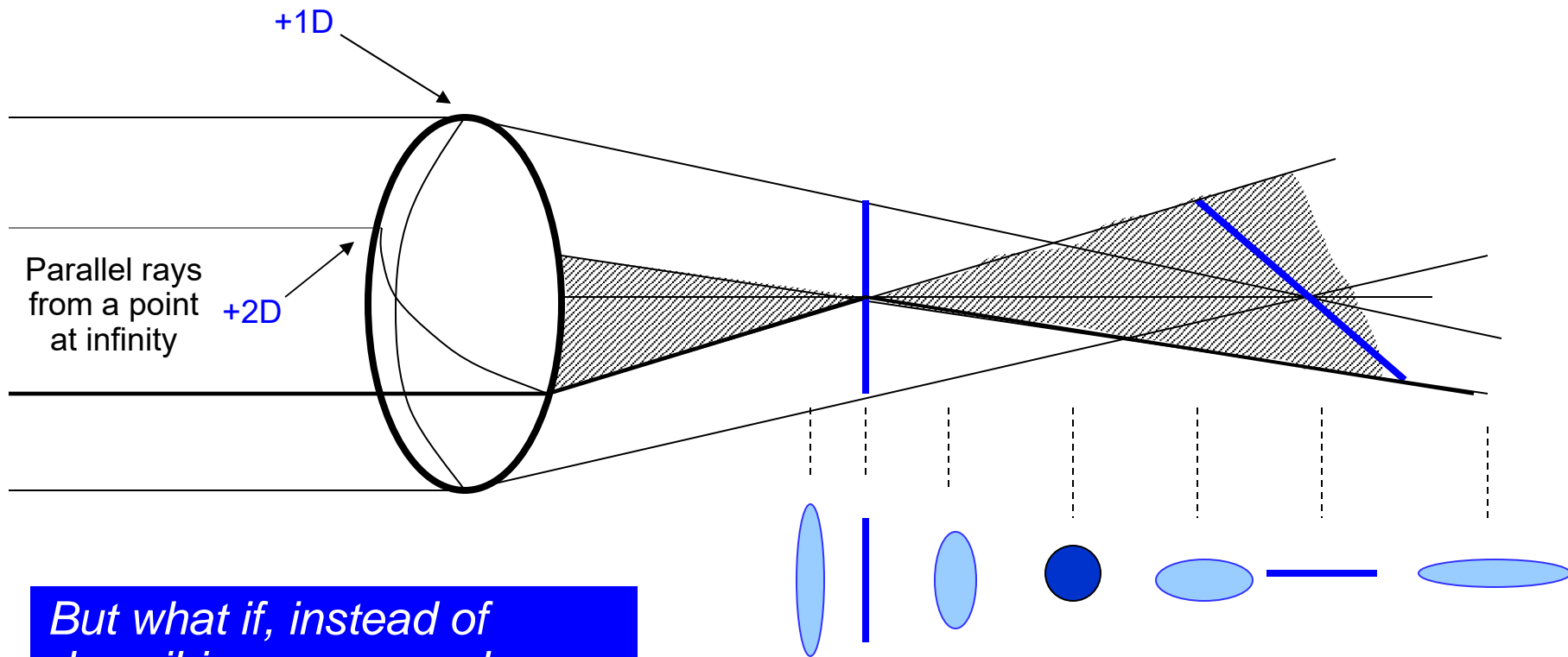
# Spherocylindrical Lenses



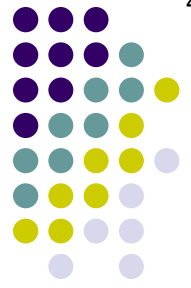
In Chapter 10, we saw that a spherocylindrical lens focuses parallel rays not to a single secondary focal point, but rather to a pair of secondary focal *lines* separated by a circle.



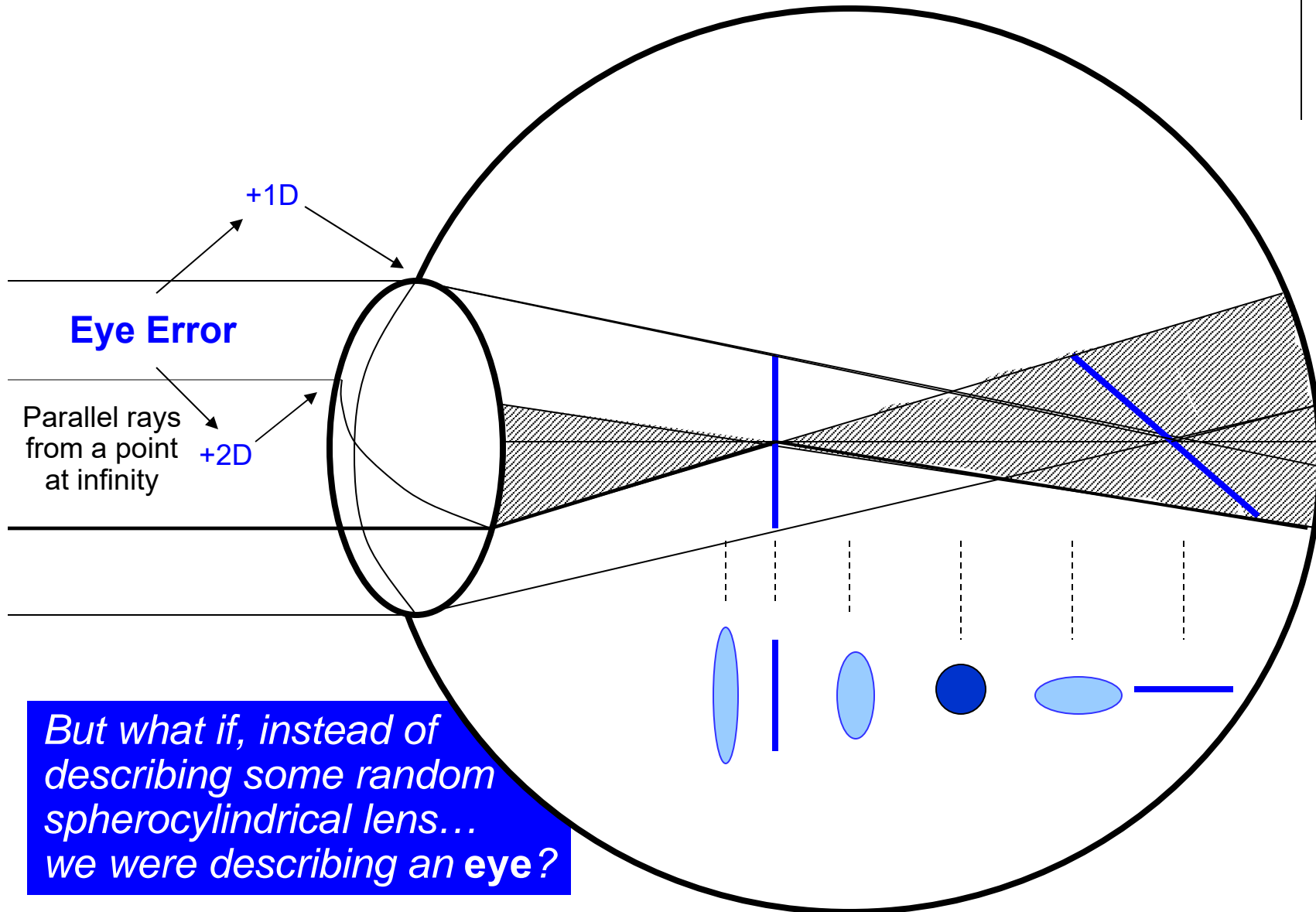
# Spherocylindrical Lenses



*But what if, instead of describing some random spherocylindrical lens...*



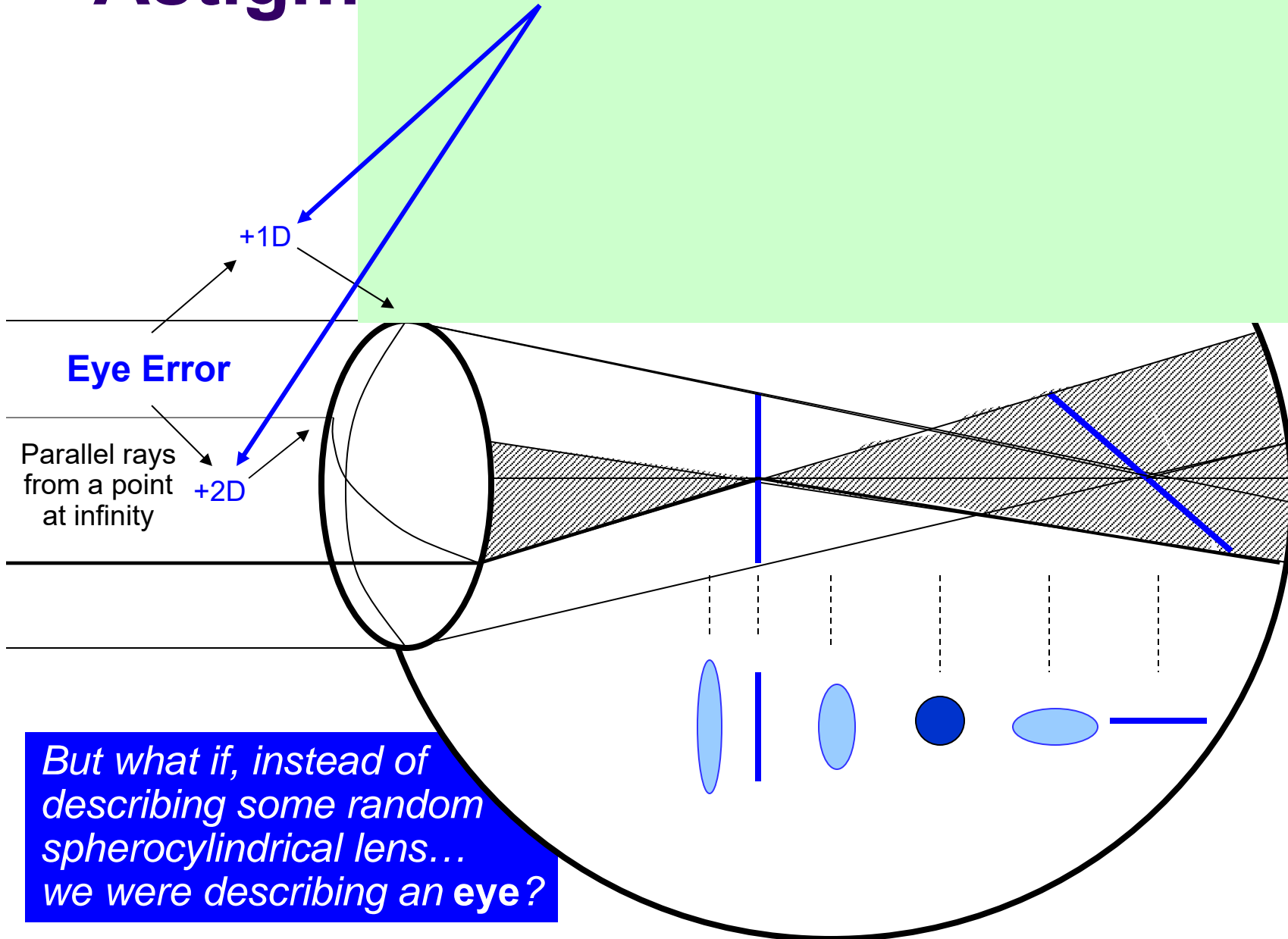
# Astigmatic Eye Error



*But what if, instead of describing some random spherocylindrical lens... we were describing an eye?*

# Astigm

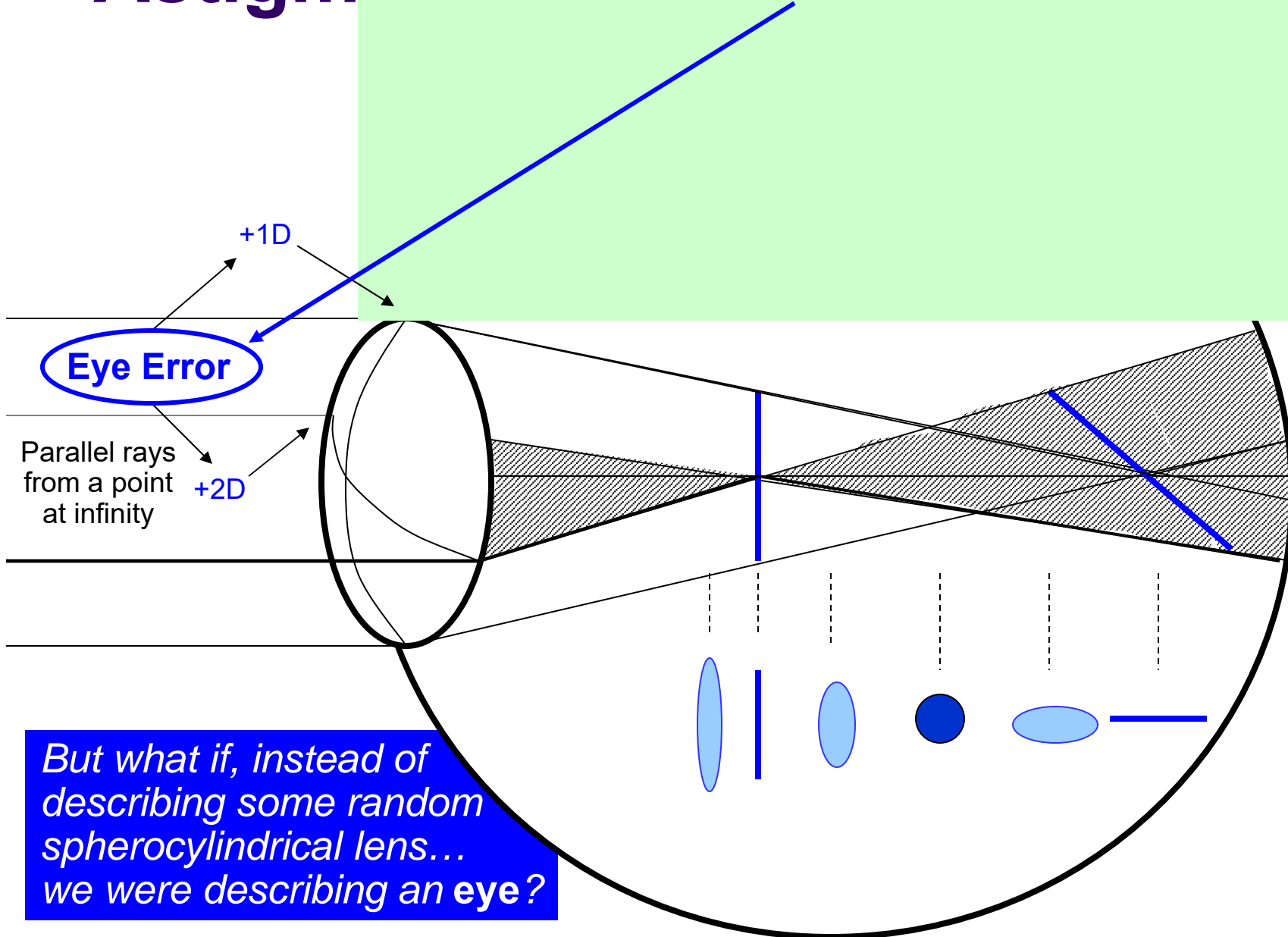
Wait a minute—a +1D lens focuses at 1 m, and a +2D at 50 cm. Shouldn't the focal lines be 50 cm and 1 m behind the cornea--not in the vitreous as shown?



*But what if, instead of describing some random spherocylindrical lens... we were describing an eye?*

# Astigm

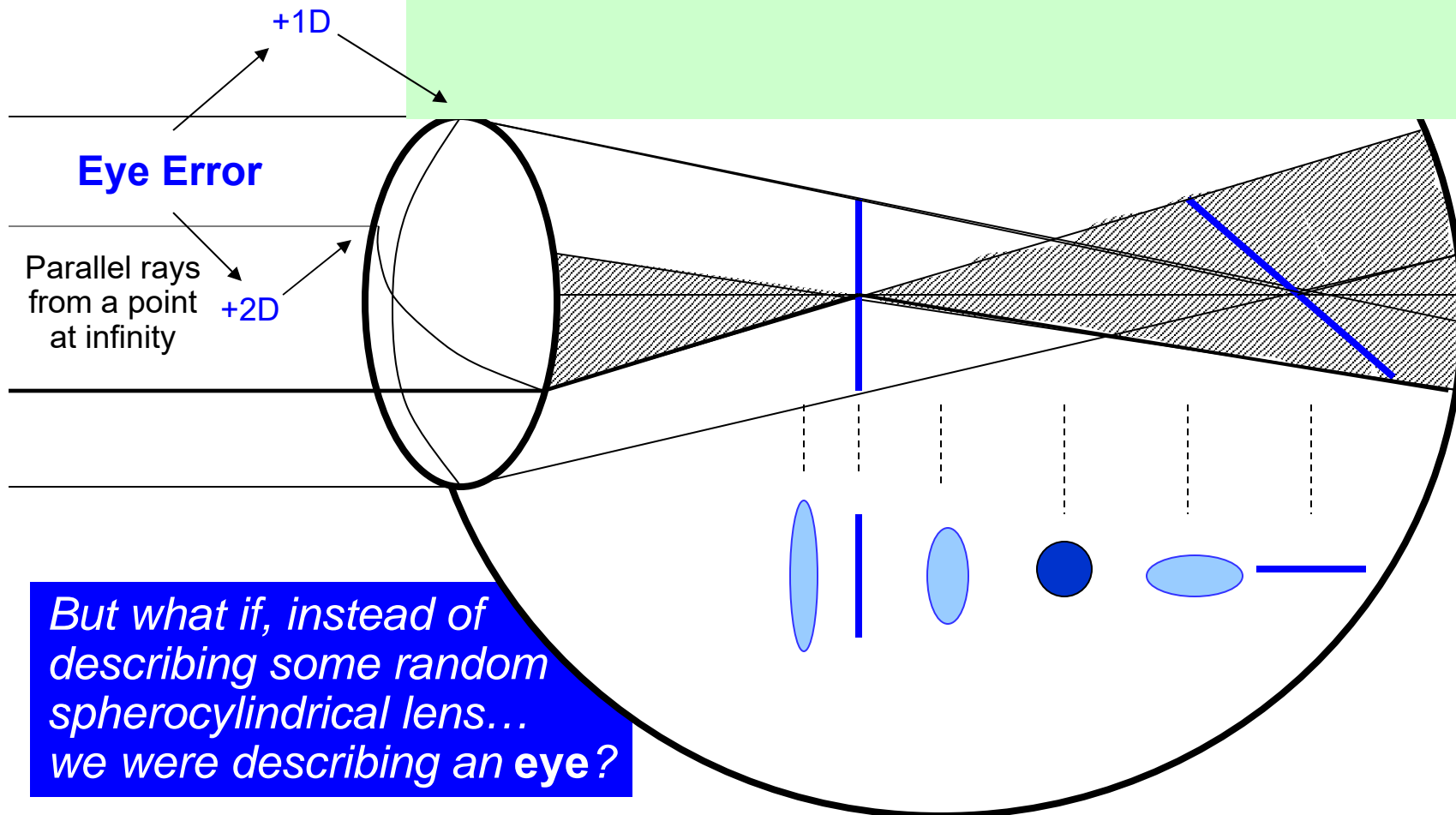
Wait a minute—a +1D lens focuses at 1 m, and a +2D at 50 cm. Shouldn't the focal lines be 50 cm and 1 m behind the cornea--not in the vitreous as shown?  
No, because we're talking about **eye error** here.



*But what if, instead of describing some random spherocylindrical lens... we were describing an eye?*

# Astigm

Wait a minute—a +1D lens focuses at 1 m, and a +2D at 50 cm. Shouldn't the focal lines be 50 cm and 1 m behind the cornea--not in the vitreous as shown? No, because we're talking about **eye error** here. Recall that 'eye error' refers to the amount of **excess** (not absolute) power in a given meridian. In the example shown, the power in the vertical meridian (i.e., with axis 180) has 1D of converging power more than is needed to put parallel rays on the retina. Likewise, the horizontal meridian (axis 090) has 2D of excess convergence.



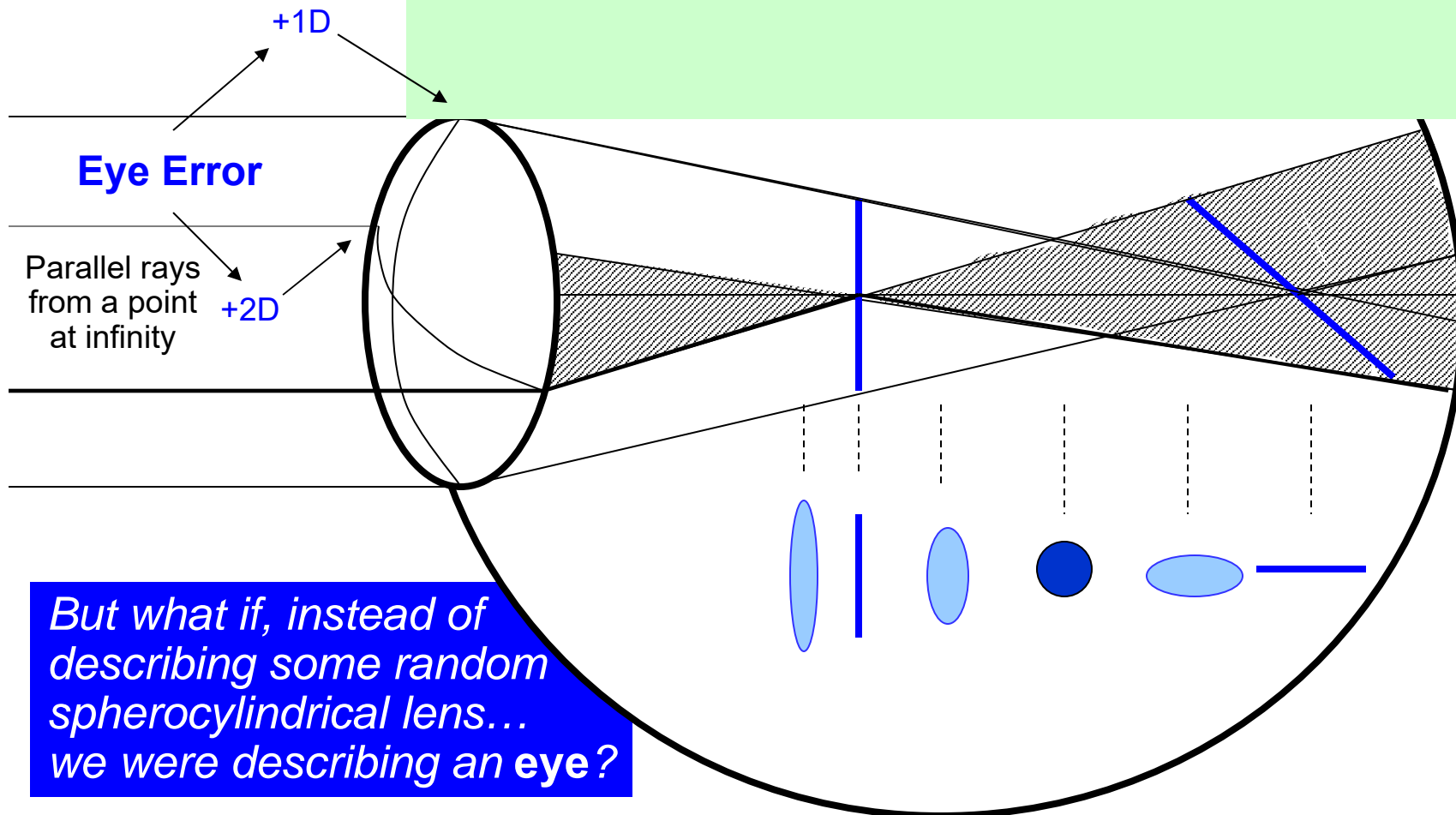
*But what if, instead of describing some random spherocylindrical lens... we were describing an eye?*

# Astigm

Wait a minute—a +1D lens focuses at 1 m, and a +2D at 50 cm. Shouldn't the focal lines be 50 cm and 1 m behind the cornea--not in the vitreous as shown?

No, because we're talking about **eye error** here. Recall that 'eye error' refers to the amount of **excess** (not absolute) power in a given meridian. In the example shown, the power in the vertical meridian (i.e., with axis 180) has 1D of converging power more than is needed to put parallel rays on the retina. Likewise, the horizontal meridian (axis 090) has 2D of excess convergence.

*So where would the focal lines be, exactly?*



*But what if, instead of describing some random spherocylindrical lens... we were describing an eye?*



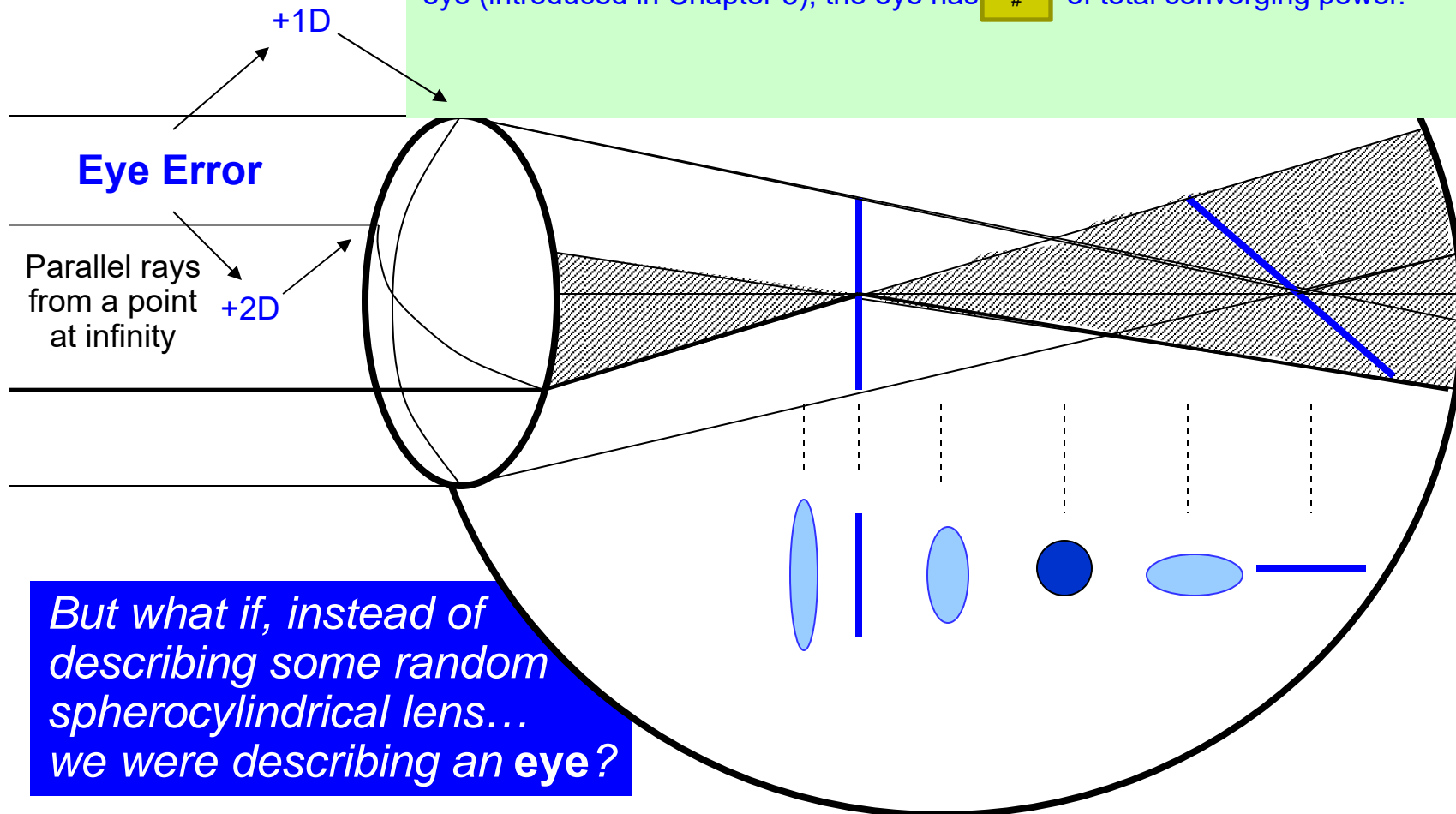
# Astigm

Wait a minute—a +1D lens focuses at 1 m, and a +2D at 50 cm. Shouldn't the focal lines be 50 cm and 1 m behind the cornea--not in the vitreous as shown?

No, because we're talking about **eye error** here. Recall that 'eye error' refers to the amount of **excess** (not absolute) power in a given meridian. In the example shown, the power in the vertical meridian (i.e., with axis 180) has 1D of converging power more than is needed to put parallel rays on the retina. Likewise, the horizontal meridian (axis 090) has 2D of excess convergence.

So where would the focal lines be, exactly?

That depends on the total power of the eye in question. In Gullstrand's reduced schematic eye (introduced in Chapter 5), the eye has # of total converging power.



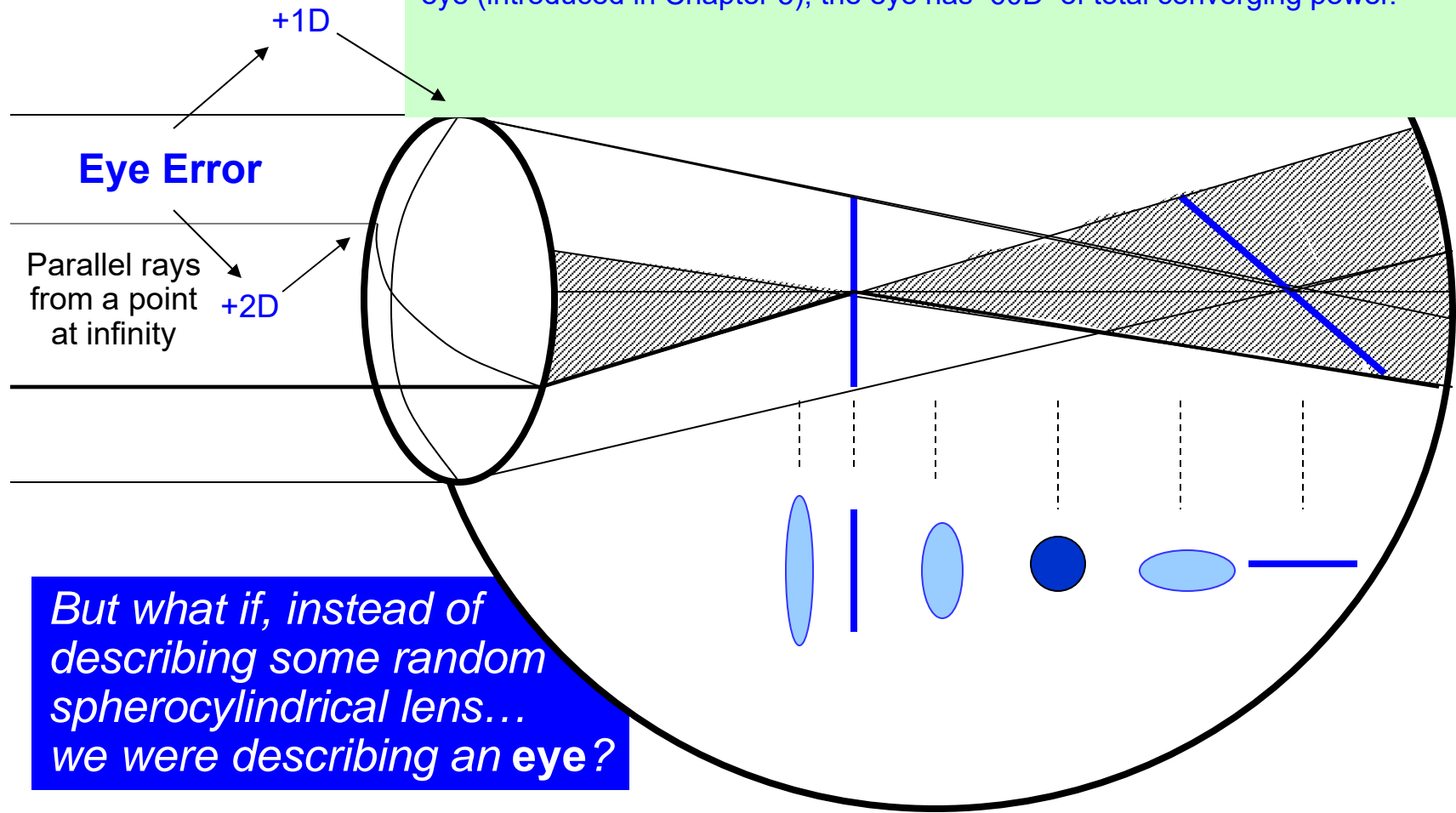
But what if, instead of describing some random spherocylindrical lens... we were describing an eye?

# Astigm

Wait a minute—a +1D lens focuses at 1 m, and a +2D at 50 cm. Shouldn't the focal lines be 50 cm and 1 m behind the cornea--not in the vitreous as shown?

No, because we're talking about **eye error** here. Recall that 'eye error' refers to the amount of **excess** (not absolute) power in a given meridian. In the example shown, the power in the vertical meridian (i.e., with axis 180) has 1D of converging power more than is needed to put parallel rays on the retina. Likewise, the horizontal meridian (axis 090) has 2D of excess convergence.

So where would the focal lines be, exactly?  
That depends on the total power of the eye in question. In Gullstrand's reduced schematic eye (introduced in Chapter 5), the eye has 60D of total converging power.



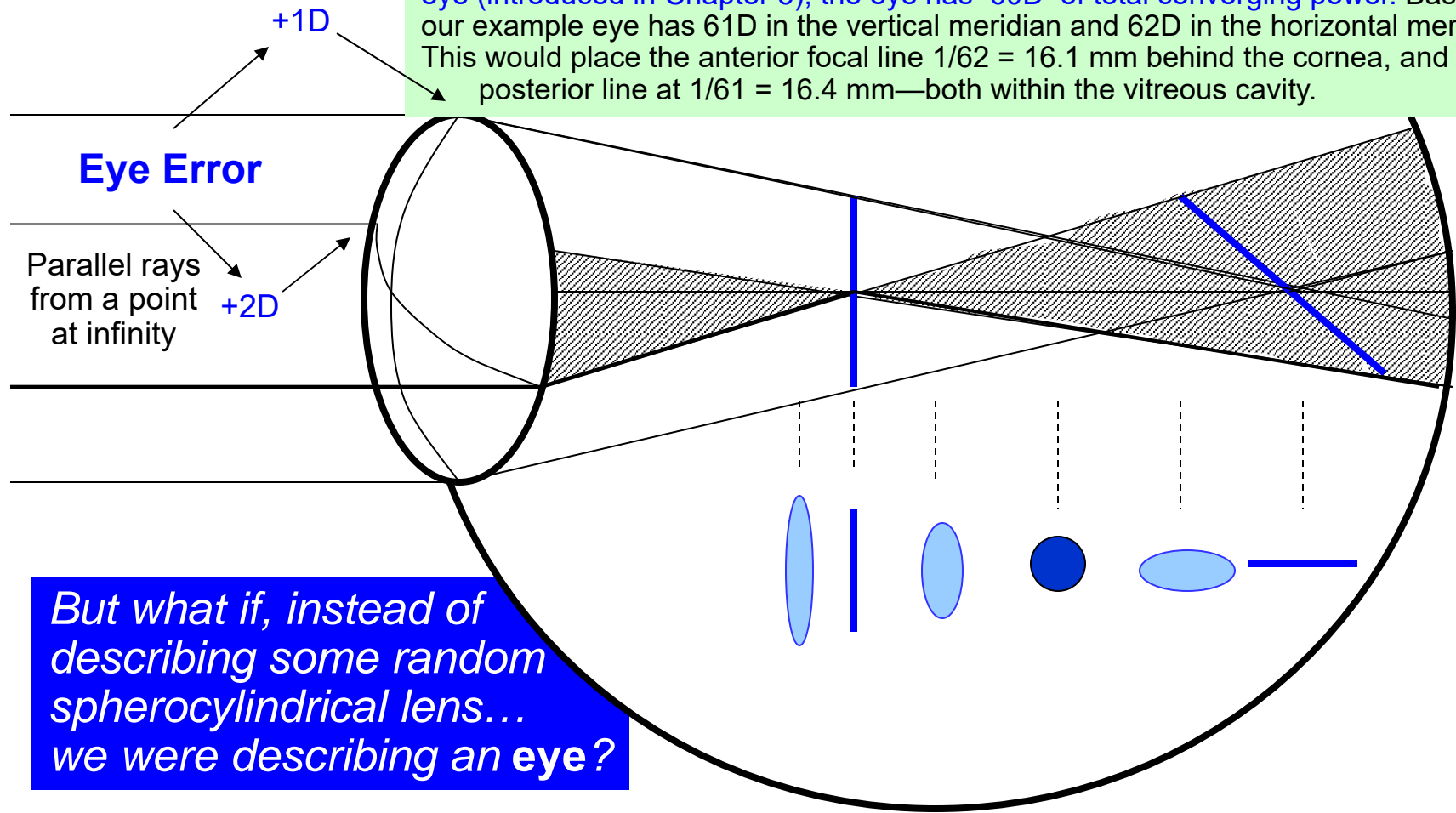
*But what if, instead of describing some random spherocylindrical lens... we were describing an eye?*

# Astigm

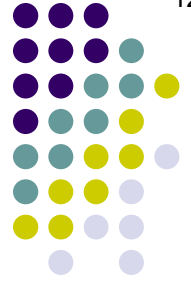
Wait a minute—a +1D lens focuses at 1 m, and a +2D at 50 cm. Shouldn't the focal lines be 50 cm and 1 m behind the cornea--not in the vitreous as shown?

No, because we're talking about **eye error** here. Recall that 'eye error' refers to the amount of **excess** (not absolute) power in a given meridian. In the example shown, the power in the vertical meridian (i.e., with axis 180) has 1D of converging power more than is needed to put parallel rays on the retina. Likewise, the horizontal meridian (axis 090) has 2D of excess convergence.

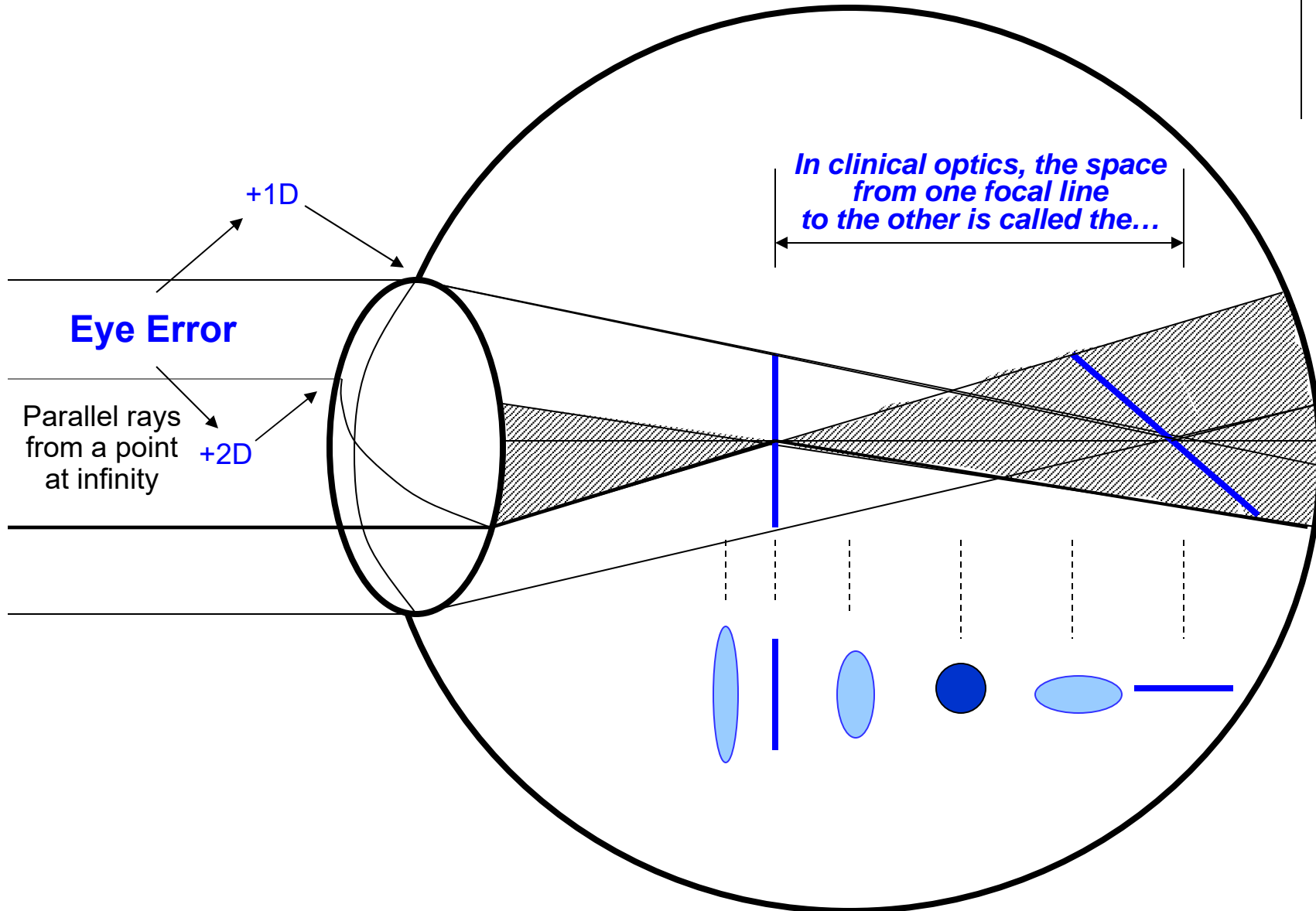
So where would the focal lines be, exactly?  
That depends on the total power of the eye in question. In Gullstrand's reduced schematic eye (introduced in Chapter 5), the eye has 60D of total converging power. Based on that, our example eye has 61D in the vertical meridian and 62D in the horizontal meridian. This would place the anterior focal line  $1/62 = 16.1$  mm behind the cornea, and the posterior line at  $1/61 = 16.4$  mm—both within the vitreous cavity.

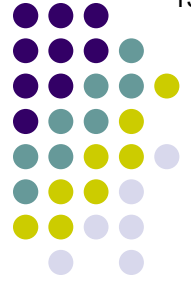


*But what if, instead of describing some random spherocylindrical lens... we were describing an eye?*

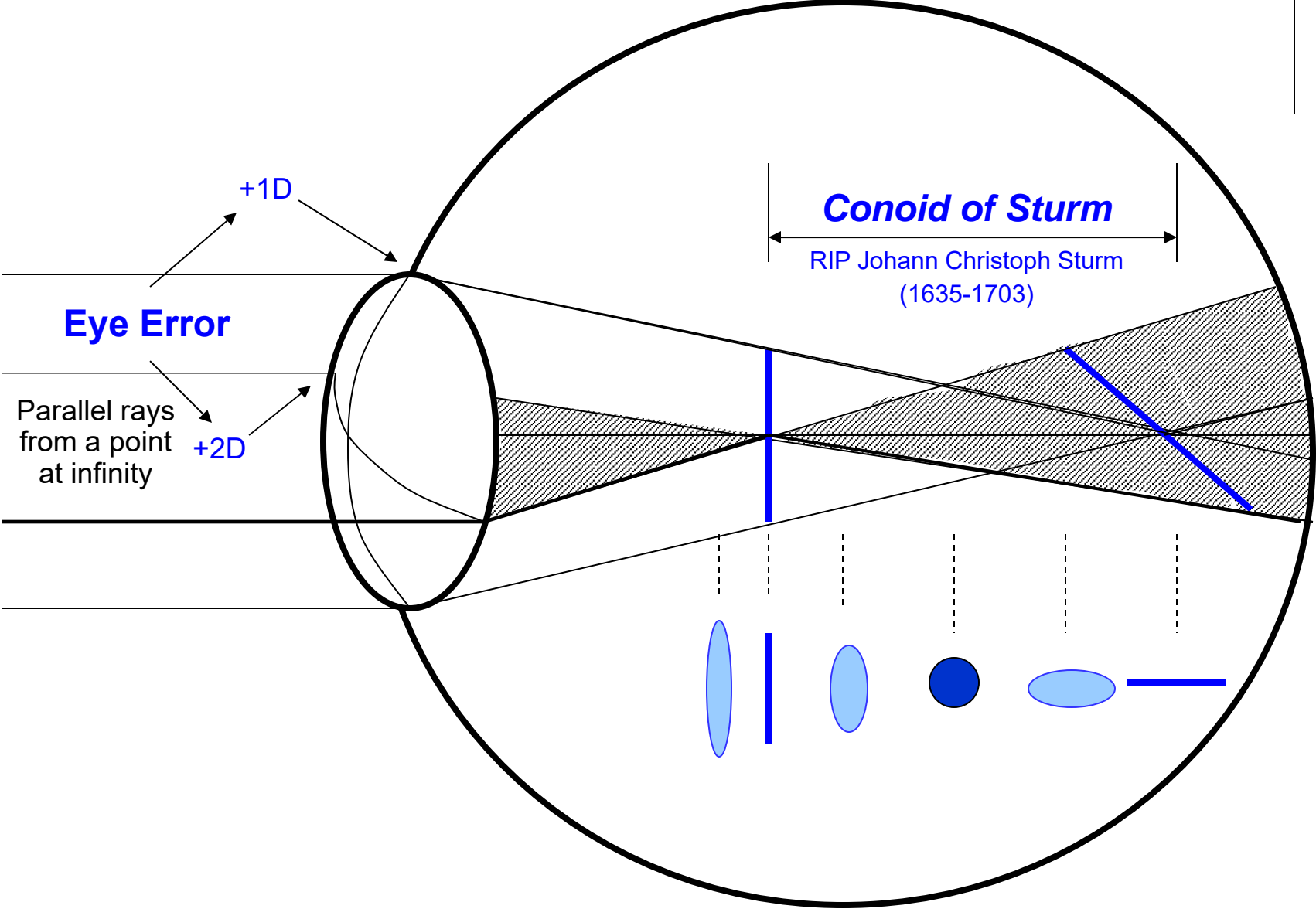


# Astigmatic Eye Error





# Astigmatic Eye Error



**Conoid of Sturm**

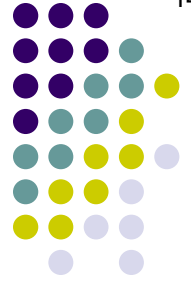
RIP Johann Christoph Sturm  
(1635-1703)

**Eye Error**

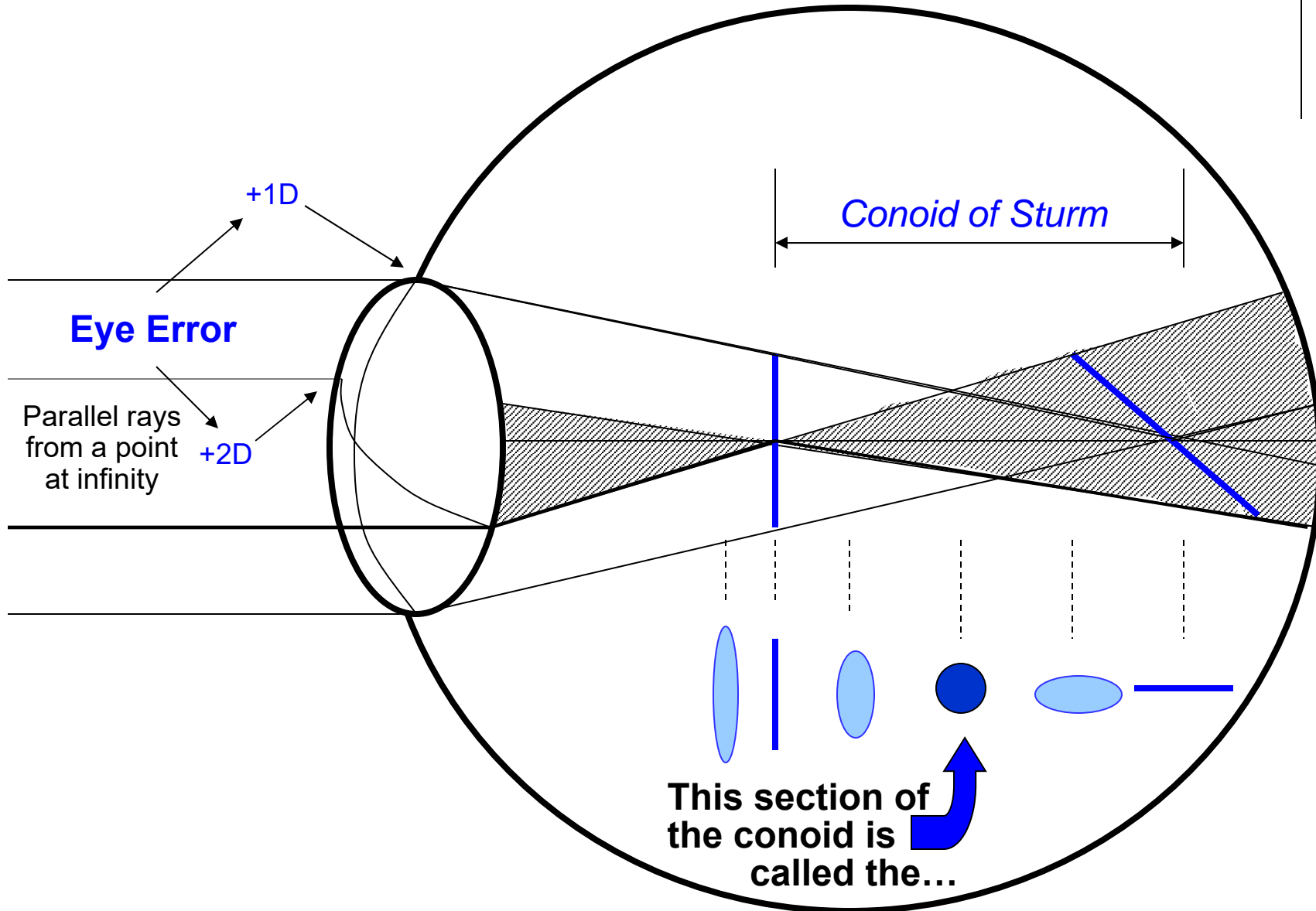
+1D

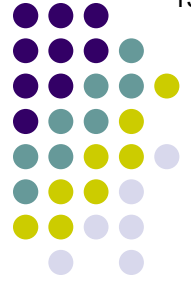
+2D

Parallel rays  
from a point  
at infinity

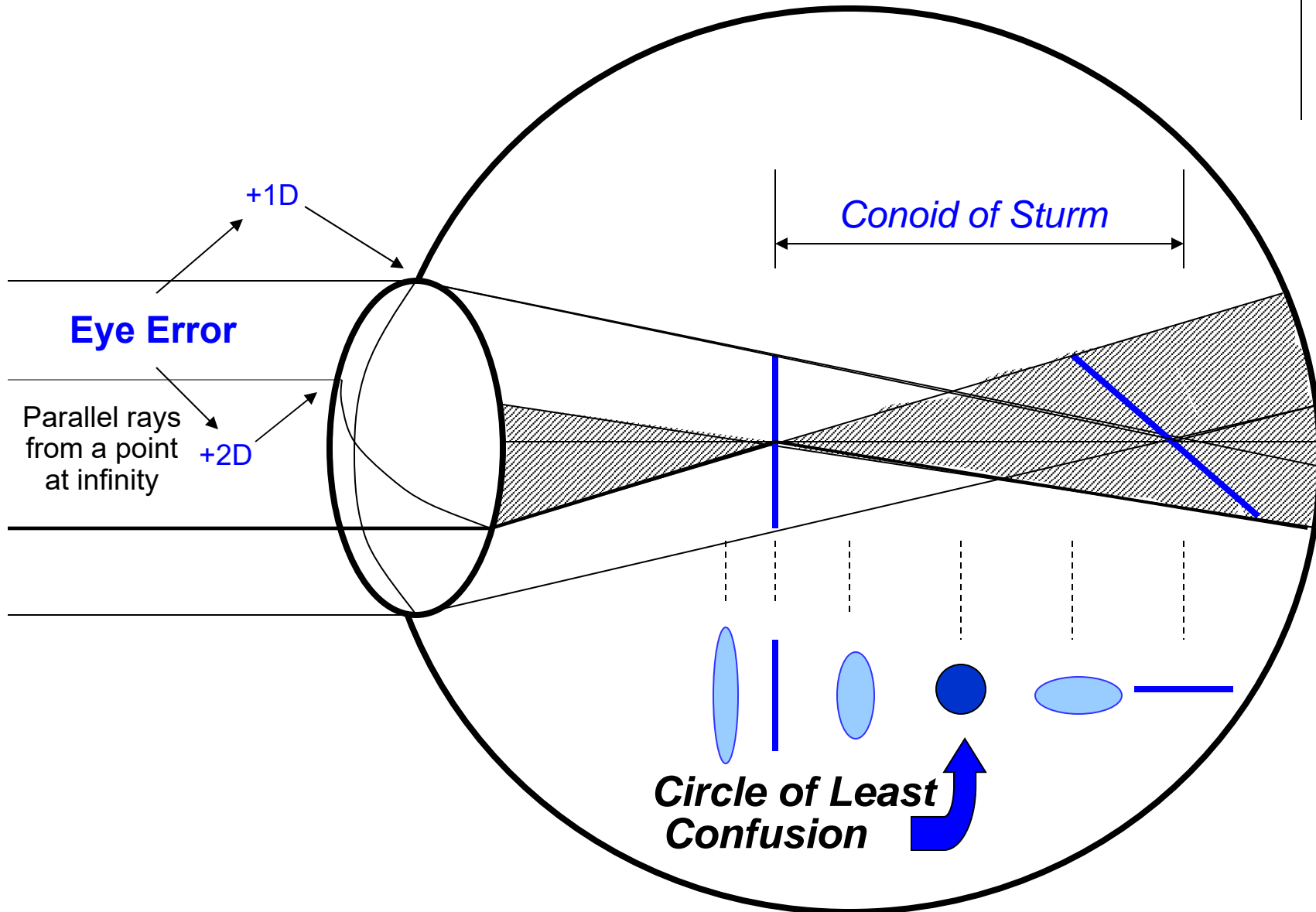


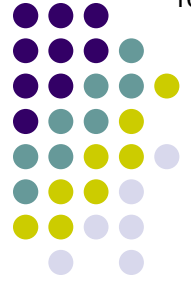
# Astigmatic Eye Error



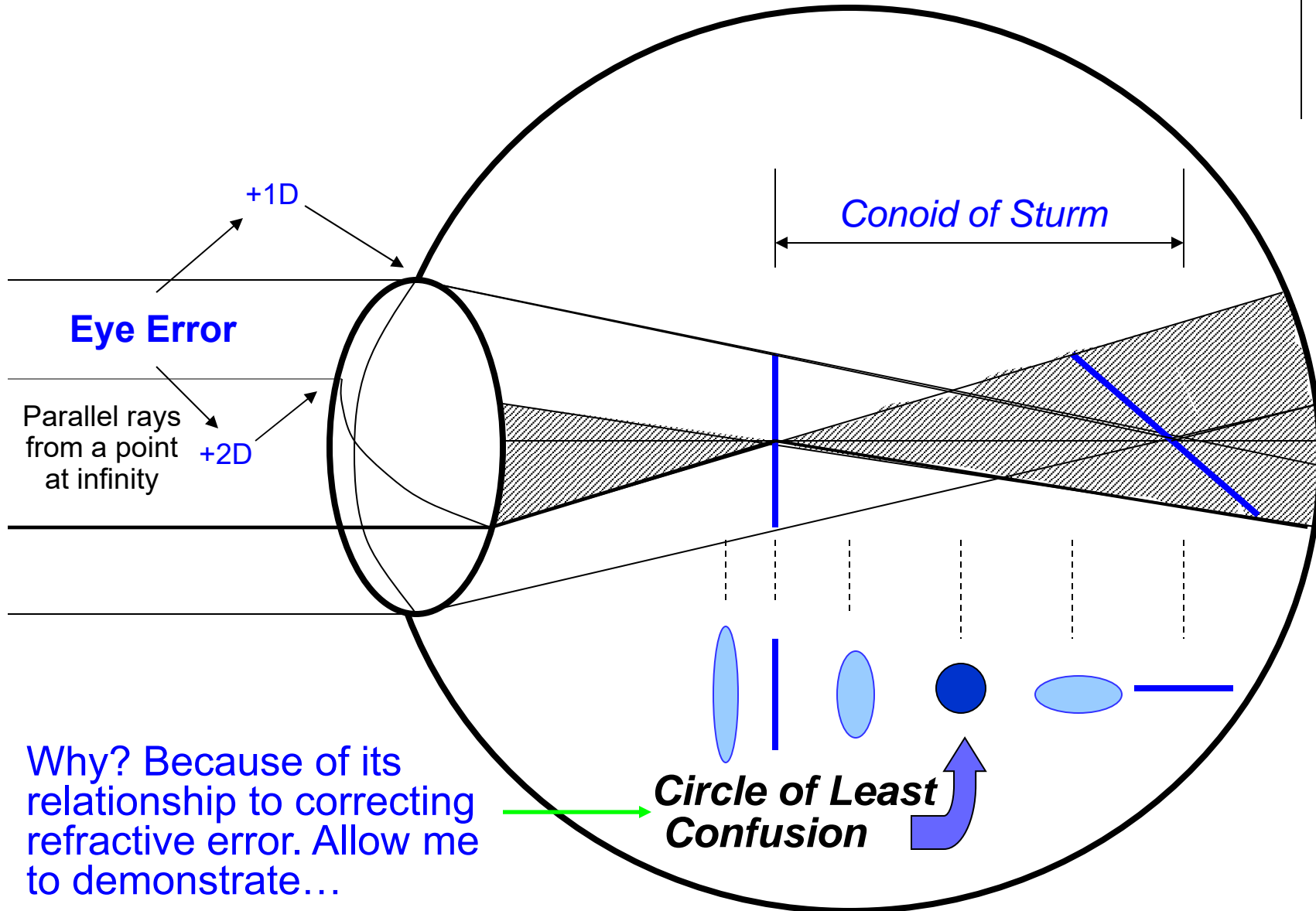


# Astigmatic Eye Error





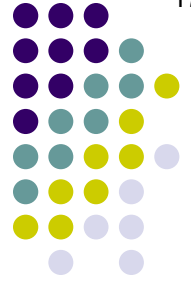
# Astigmatic Eye Error



Why? Because of its relationship to correcting refractive error. Allow me to demonstrate...

**Circle of Least Confusion**

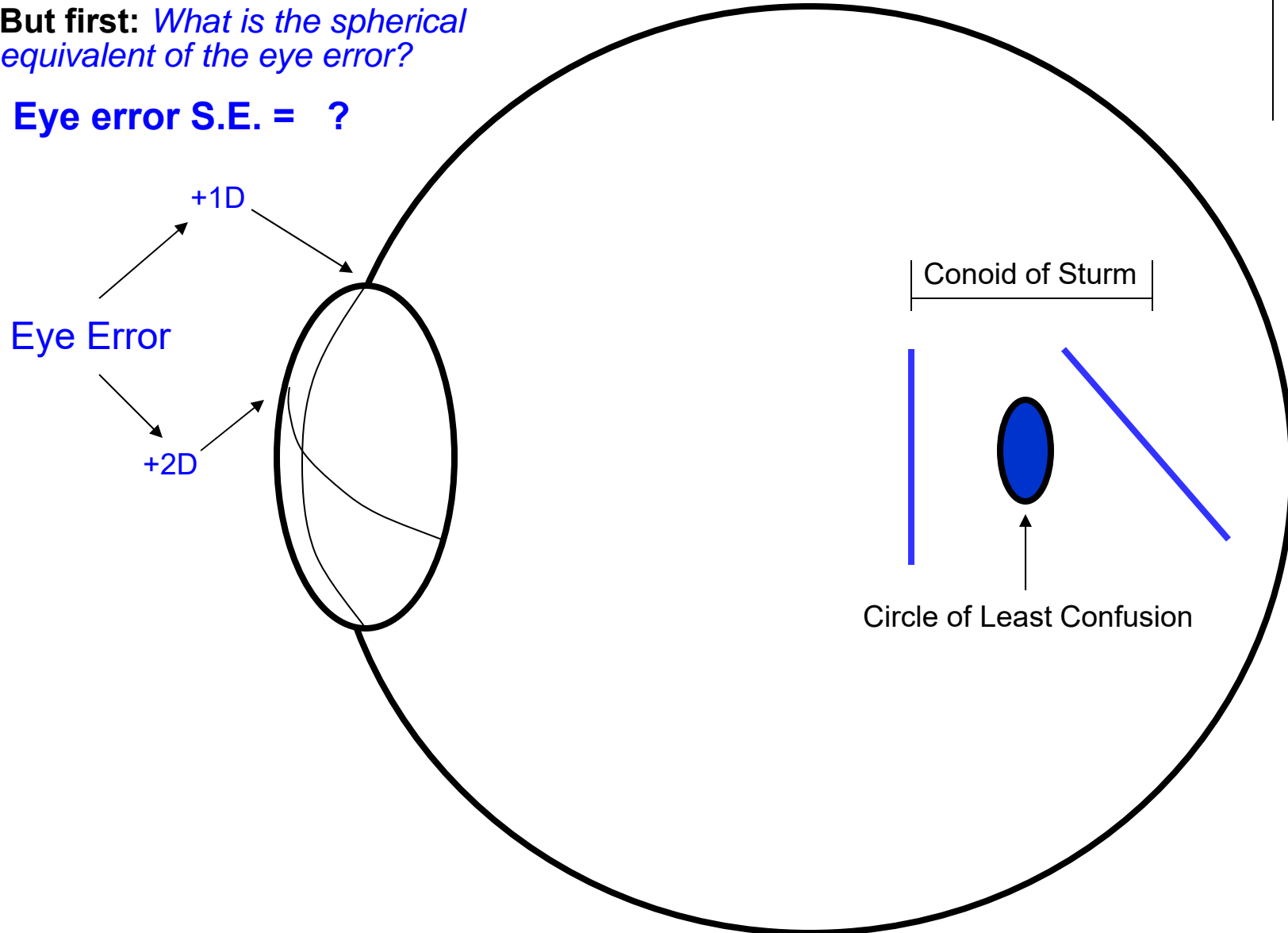


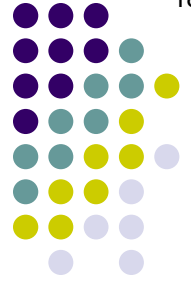


# Astigmatic Eye Error

But first: *What is the spherical equivalent of the eye error?*

Eye error S.E. = ?

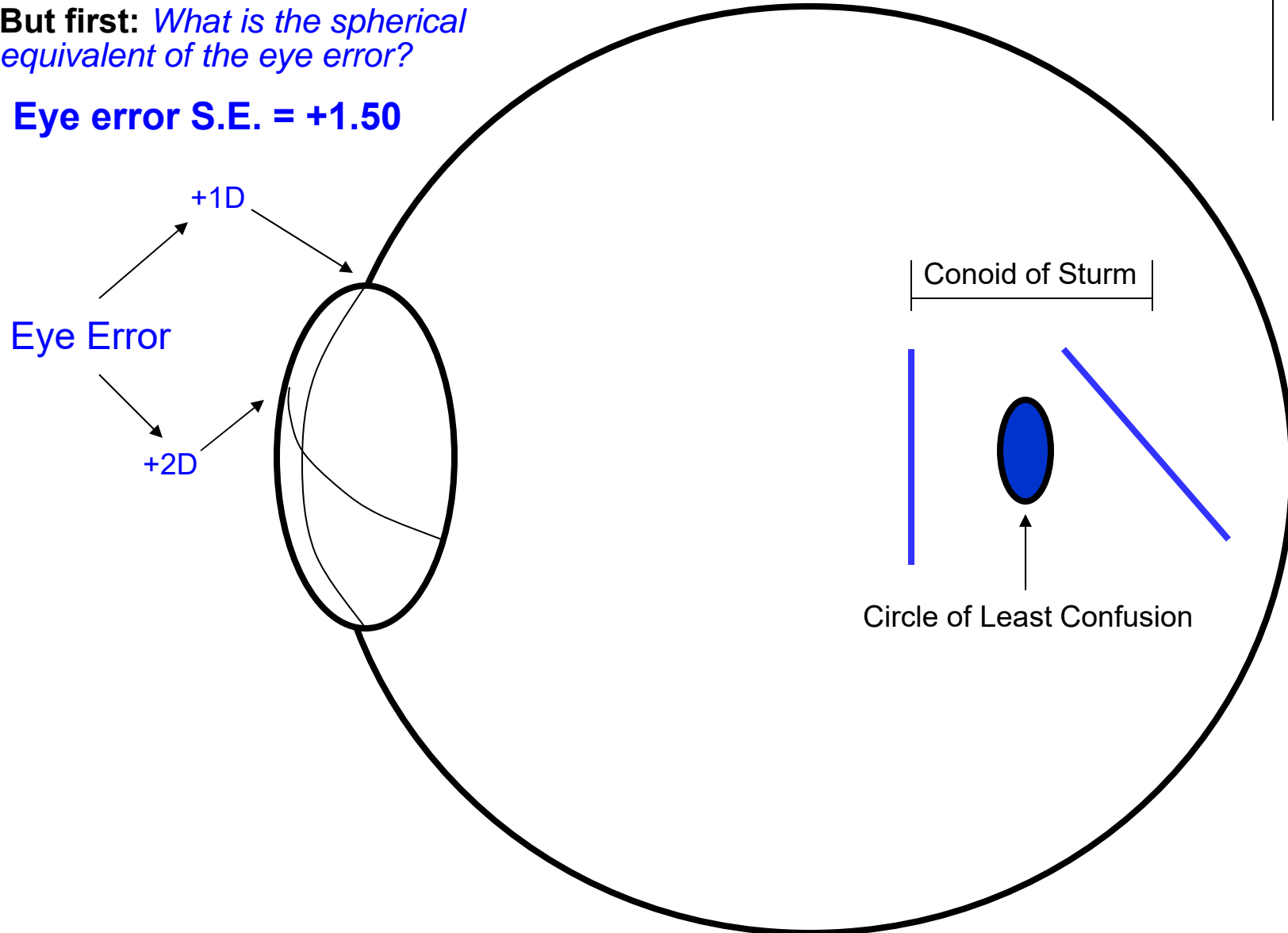


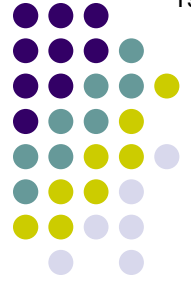


# Astigmatic Eye Error

But first: *What is the spherical equivalent of the eye error?*

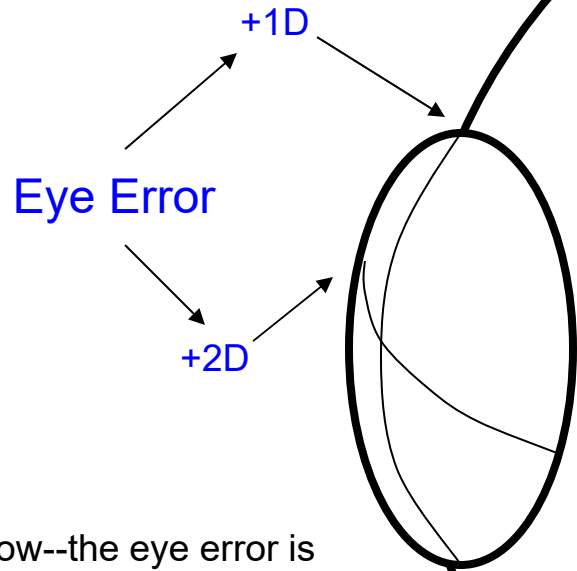
Eye error S.E. = +1.50



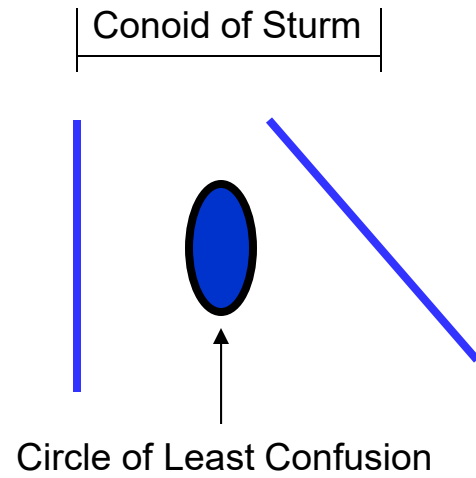


# Astigmatic Eye Error

Eye error S.E. = +1.50



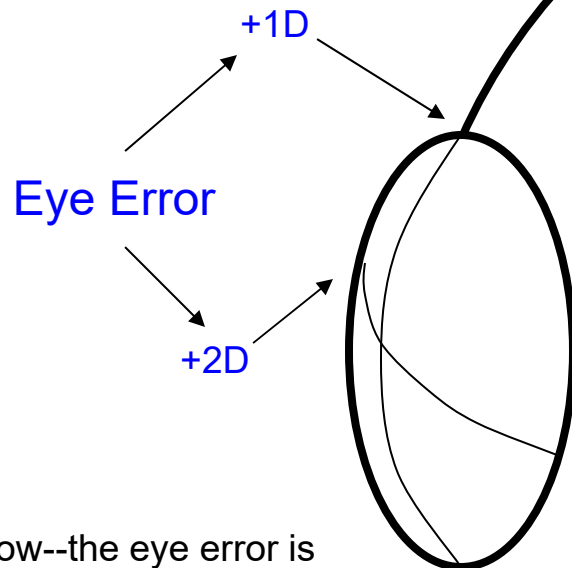
Now--the eye error is  
**+1 x 180**  
**+2 x 090**  
 Ignoring vertex distance,  
 what is the refractive correction,  
 and what is the S.E. of that  
 correction?



# Astigmatic Eye Error



Eye error S.E. = +1.50



Now--the eye error is

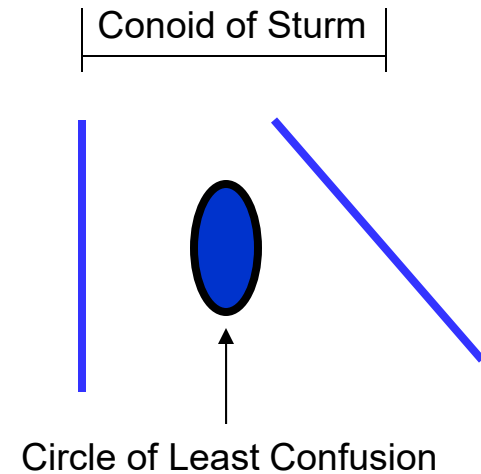
**+1 x 180**

**+2 x 090**

Ignoring vertex distance, the  
corrective lenses needed are:

**-1 x 180** (to offset the +1 x 180)

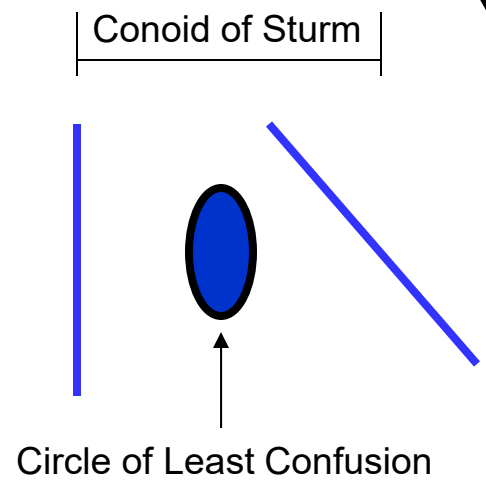
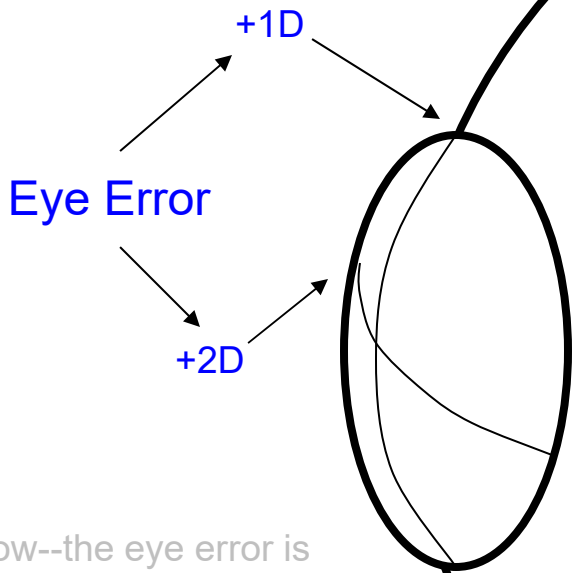
**-2 x 090** (to offset the +2 x 090)





# Astigmatic Eye Error

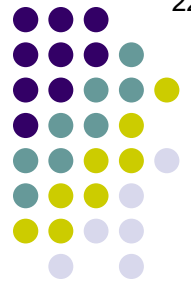
Eye error S.E. = +1.50



Now--the eye error is  
 +1 x 180  
 +2 x 090  
 Ignoring vertex distance, the  
 corrective lenses needed are:  
 -1 x 180 (to offset the +1 x 180)  
 -2 x 090 (to offset the +2 x 090)

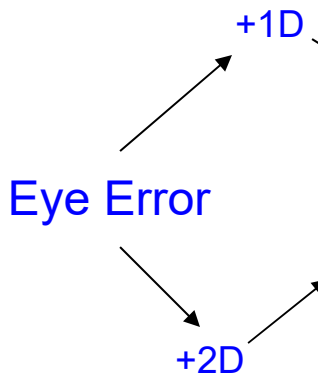
**-1 x 180**  
**-2 x 090**

Note that these two corrective lenses have an average power of **-1.50**



# Astigmatic Eye Error

Eye error S.E. = +1.50



Conoid of Sturm

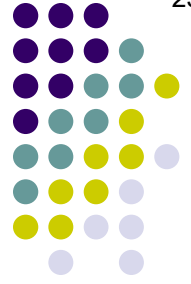
Note that the 'average corrective lens' (ie, the S.E. of the corrective cylinders) is equal in power (but opposite in sign) to the 'average eye error' (ie, the S.E. of the spherocylindrical error lens).

Circle of Least Confusion

Now--the eye error is  
 +1 x 180  
 +2 x 090  
 Ignoring vertex distance, the corrective lenses needed are:  
 -1 x 180 (to offset the +1 x 180)  
 -2 x 090 (to offset the +2 x 090)

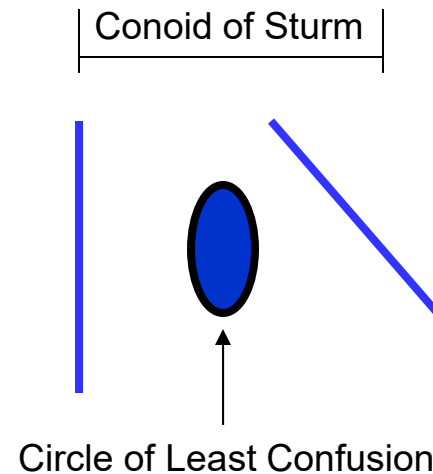
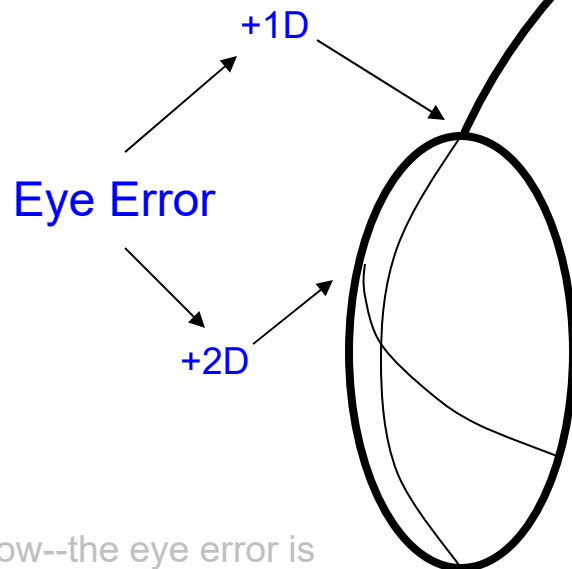
**-1 x 180**  
**-2 x 090**

Note that these two corrective lenses have an average power of **-1.50**



# Astigmatic Eye Error

Eye error S.E. = +1.50



Now--the eye error is  
 +1 x 180  
 +2 x 090  
 Ignoring vertex distance, the  
 corrective lenses needed are:  
 -1 x 180 (to offset the +1 x 180)  
 -2 x 090 (to offset the +2 x 090)

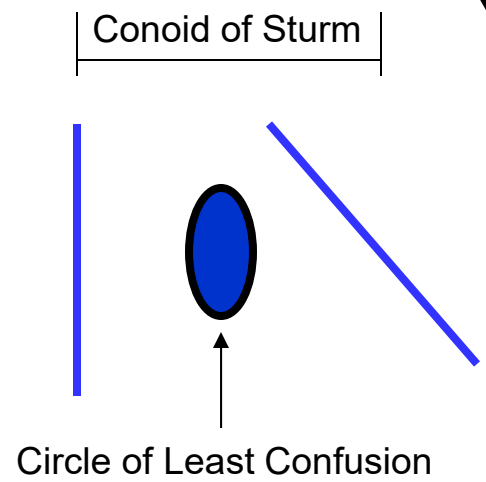
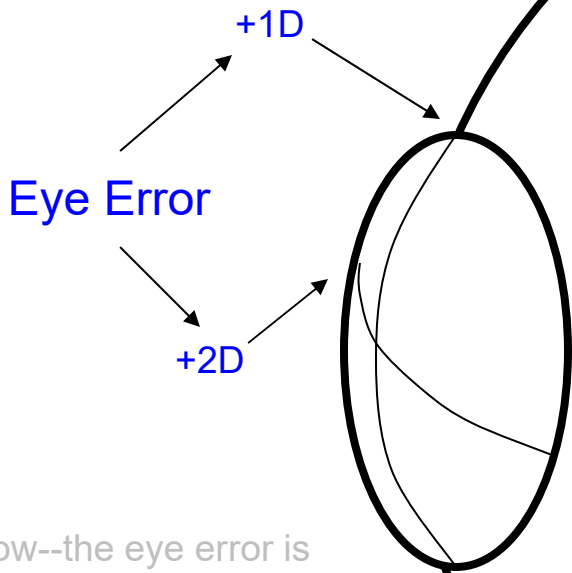
Or, when written like an Rx for glasses:  
 -1 -1 x 090 (or -2 +1 x 180).

You may not know how to do this yet.  
 Don't worry; we'll get to it soon enough.  
 For now, just trust me.



# Astigmatic Eye Error

Eye error S.E. = +1.50



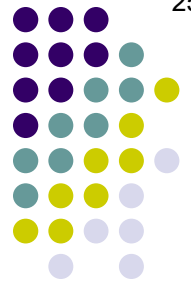
Now--the eye error is  
 +1 x 180  
 +2 x 090  
 Ignoring vertex distance, the  
 corrective lenses needed are:  
 -1 x 180 (to offset the +1 x 180)  
 -2 x 090 (to offset the +2 x 090)

Or, when written like an Rx for glasses:  
 -1 -1 x 090 (or -2 +1 x 180).

The S.E. of either Rx is the same: **-1.50.**

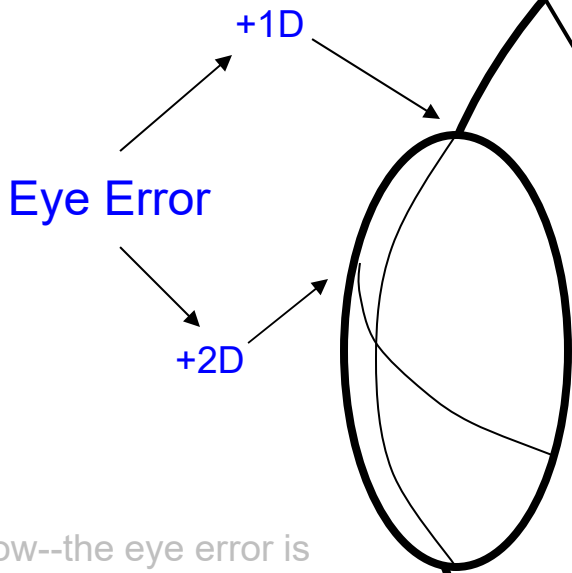
← Ditto. Again, trust me.





# Astigmatic Eye Error

Eye error S.E. = +1.50

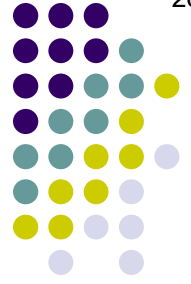


Conoid of Sturm

Again we see that the 'average corrective lens' (the S.E. of the glasses correction) is equal in power (but opposite in sign) to the 'average eye error' (ie, the S.E. of the spherocylindrical error lens). Hmm...  
***I wonder:***

Now--the eye error is  
 +1 x 180  
 +2 x 090  
 Ignoring vertex distance, the corrective lenses needed are:  
 -1 x 180 (to offset the +1 x 180)  
 -2 x 090 (to offset the +2 x 090)

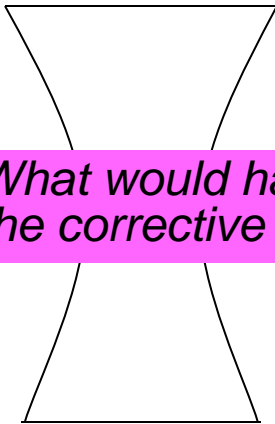
Or, when written like an Rx for glasses:  
 -1 -1 x 090 (or -2 +1 x 180).  
 The S.E. of either Rx is the same: **-1.50.**



# Astigmatic Eye Error

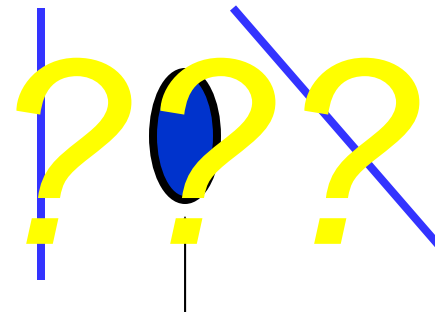
Eye error S.E. = +1.50

-1.50D lens



*What would happen to the conoid if we put the corrective S.E. in front of this eye?*

Conoid of Sturm



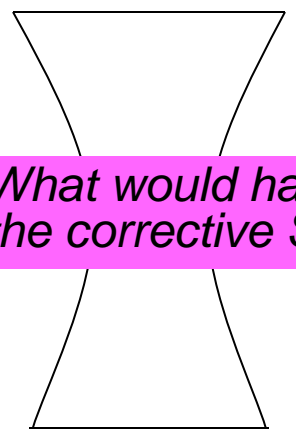
Circle of Least Confusion



# Astigmatic Eye Error

Eye error S.E. = +1.50

-1.50D lens



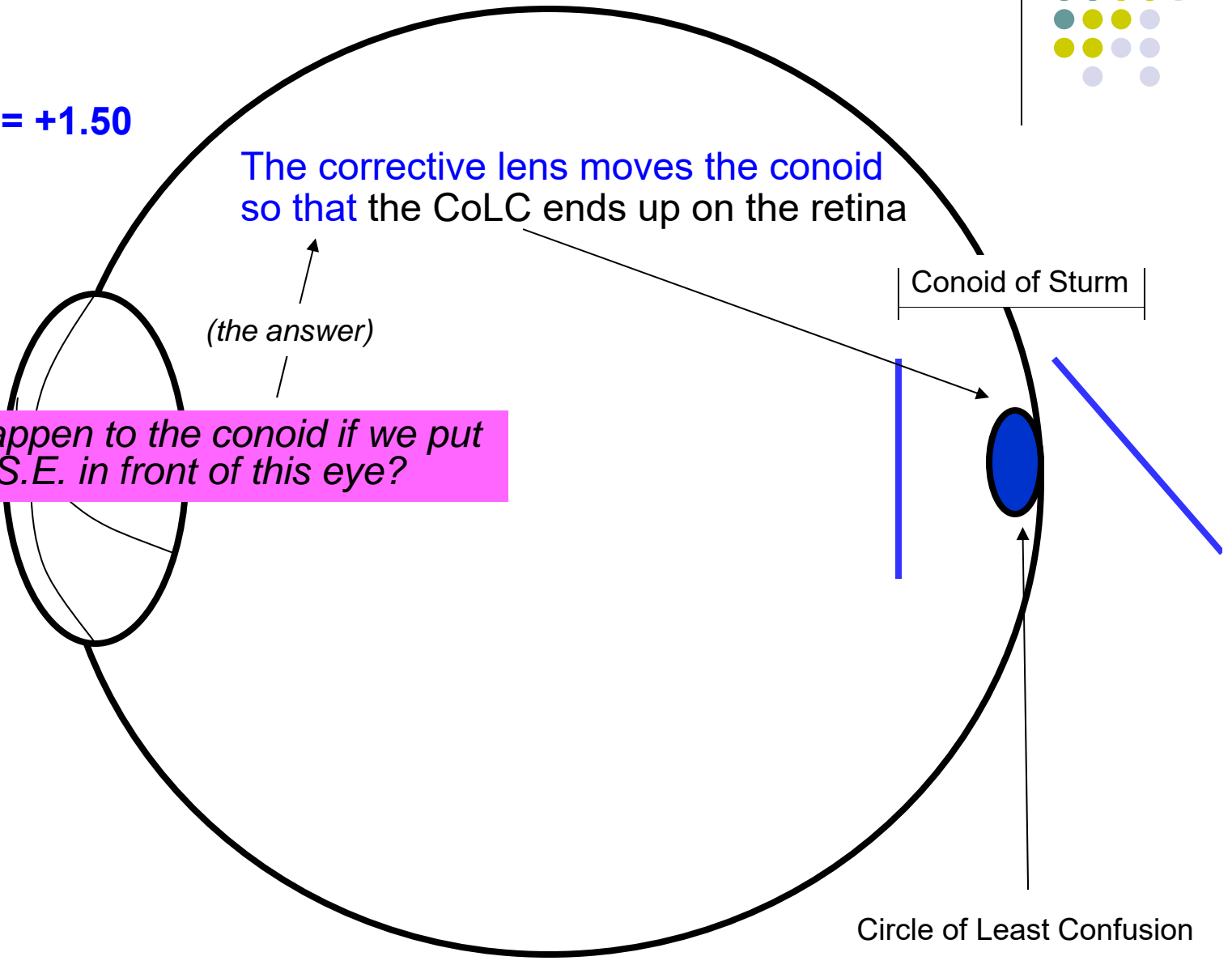
The corrective lens moves the conoid so that the CoLC ends up on the retina

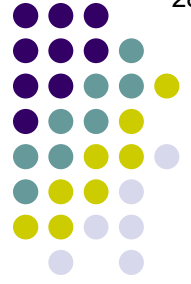
(the answer)

What would happen to the conoid if we put the corrective S.E. in front of this eye?

Conoid of Sturm

Circle of Least Confusion

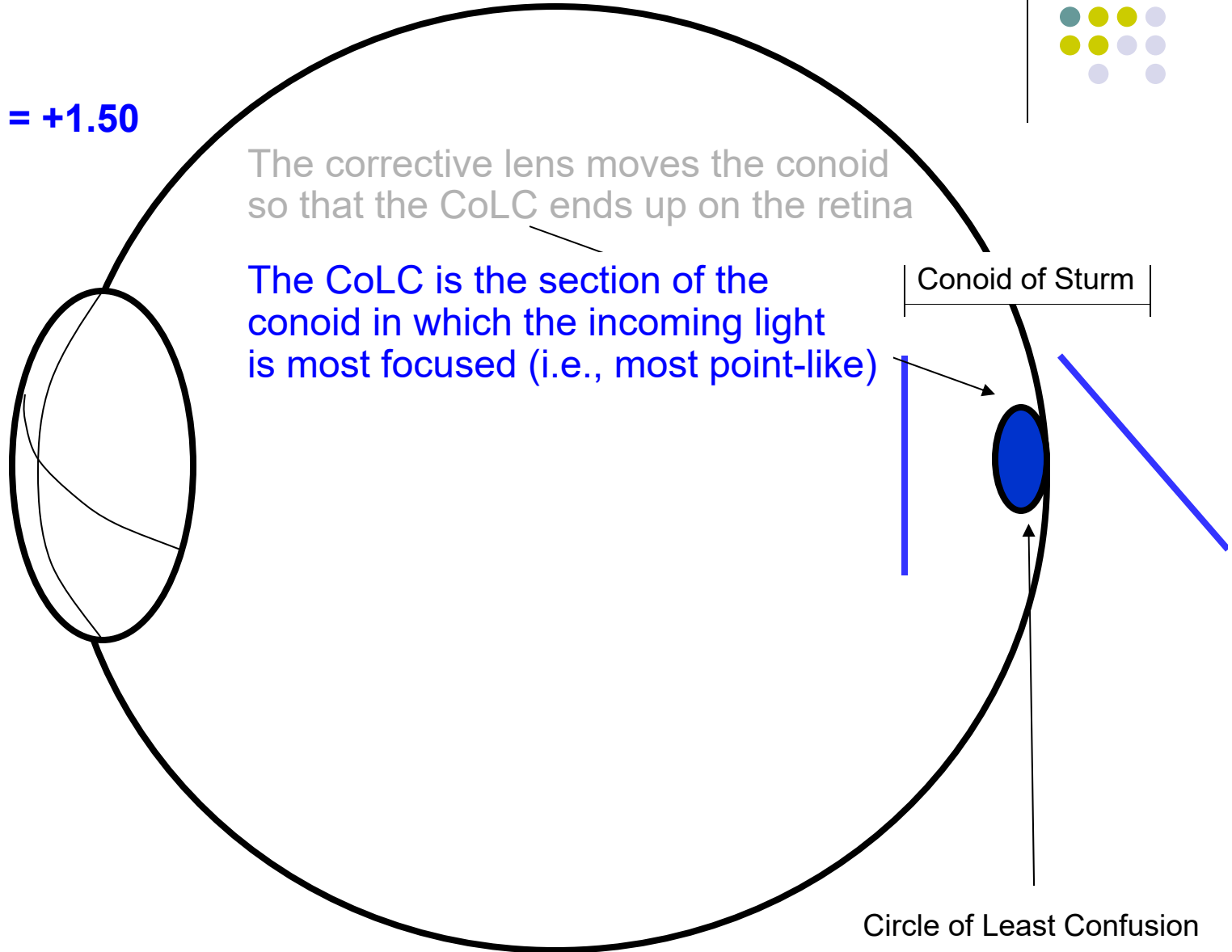
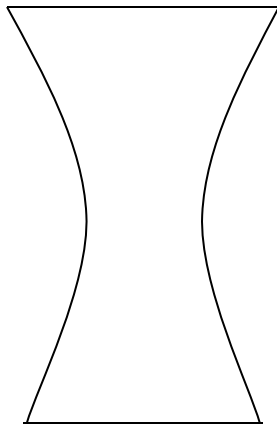




# Astigmatic Eye Error

Eye error S.E. = +1.50

-1.50D lens



The corrective lens moves the conoid so that the CoLC ends up on the retina

The CoLC is the section of the conoid in which the incoming light is most focused (i.e., most point-like)

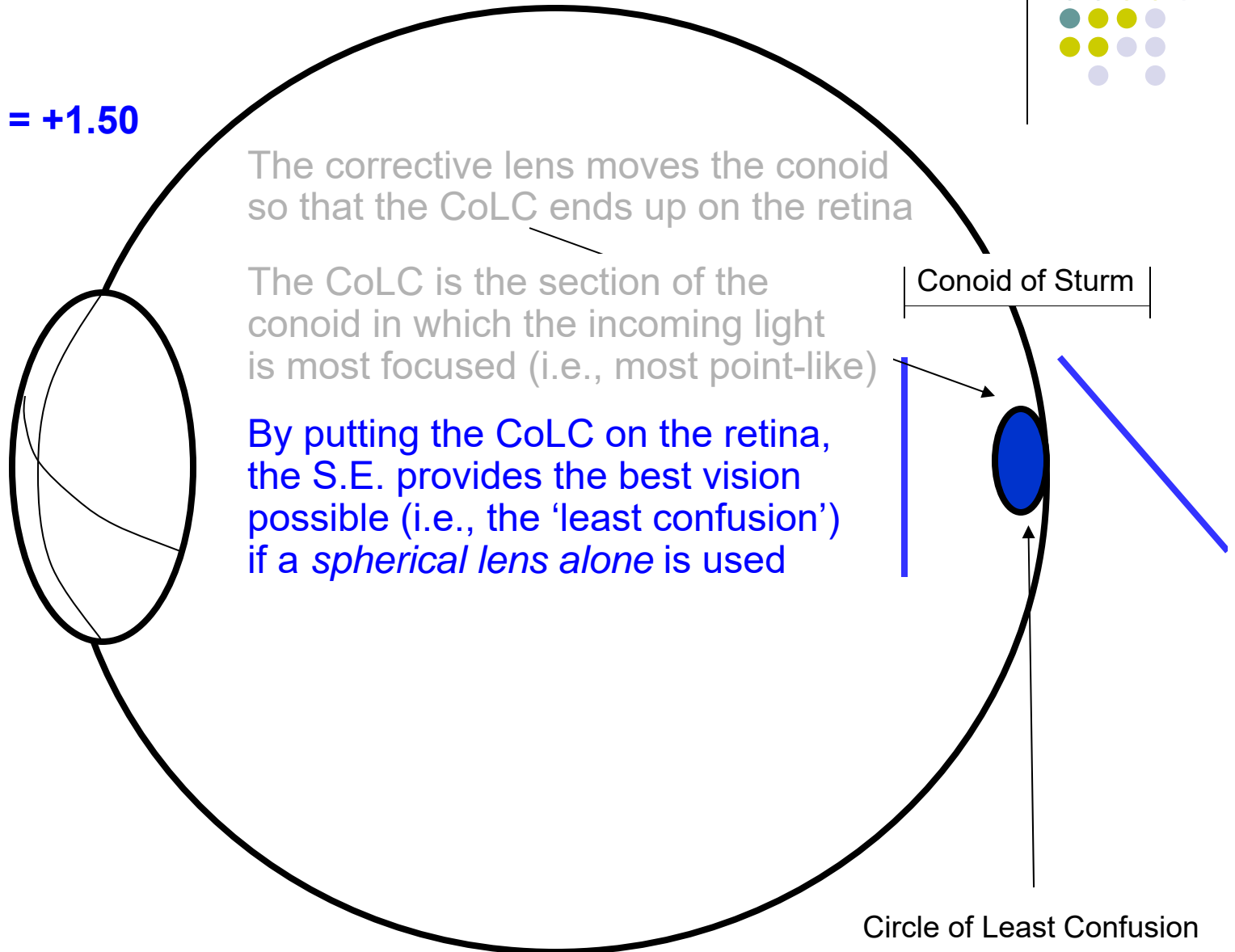
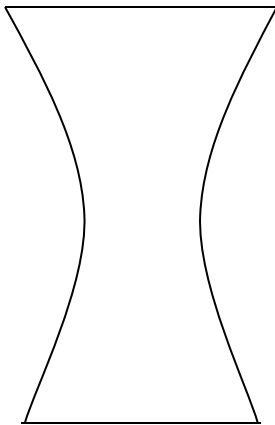
Conoid of Sturm

Circle of Least Confusion

# Astigmatic Eye Error

Eye error S.E. = +1.50

-1.50D lens

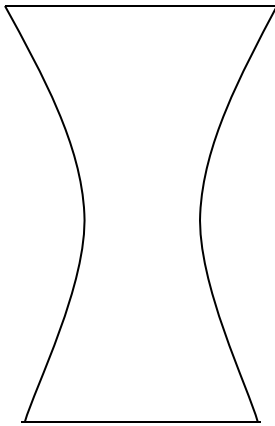


# Astigmatic Eye Error



Eye error S.E. = +1.50

-1.50D lens



The corrective lens moves the conoid so that the CoLC ends up on the retina

The CoLC is the section of the conoid in which the incoming light is most focused (i.e., most point-like)

By putting the CoLC on the retina, the S.E. provides the best vision possible (i.e., the 'least confusion') if a *spherical lens alone* is used

\* The smaller the CoLC, the better the vision

Conoid of Sturm

Circle of Least Confusion

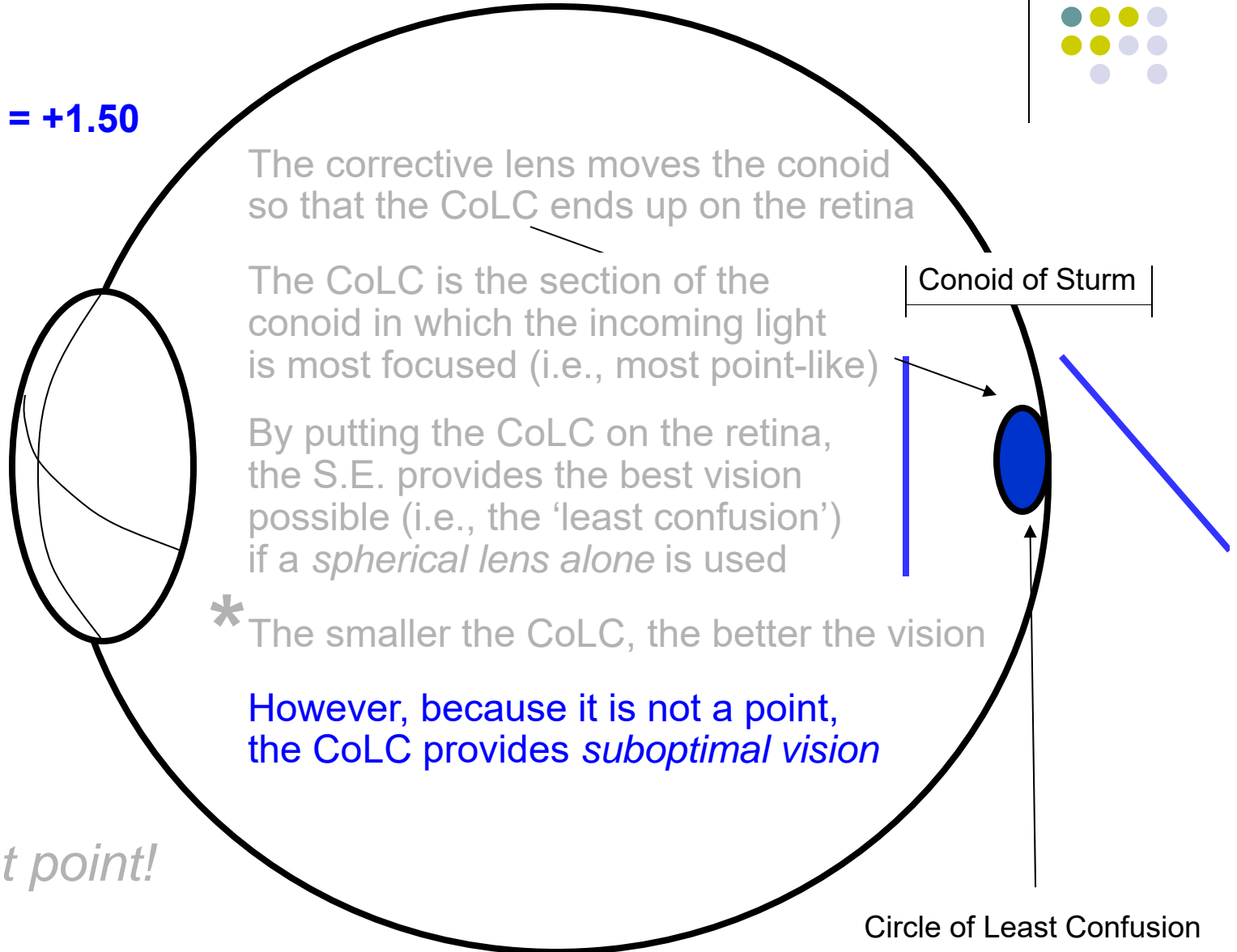
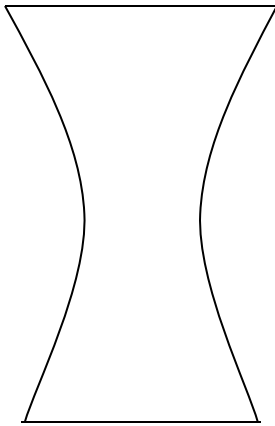
\* *Important point!*

# Astigmatic Eye Error



Eye error S.E. = +1.50

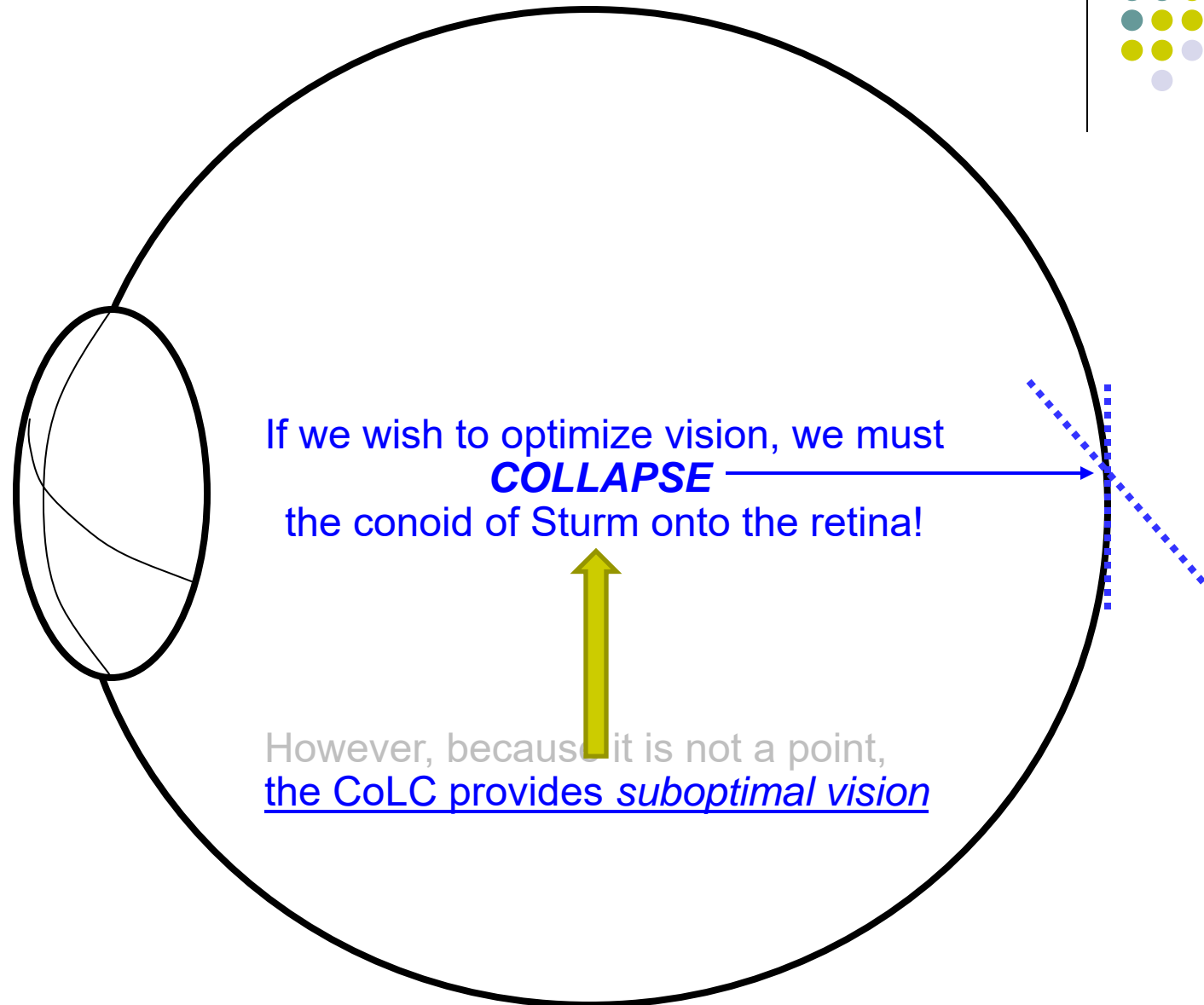
-1.50D lens



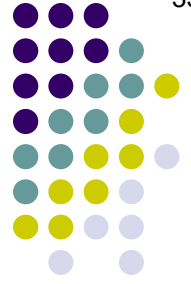
\* *Important point!*



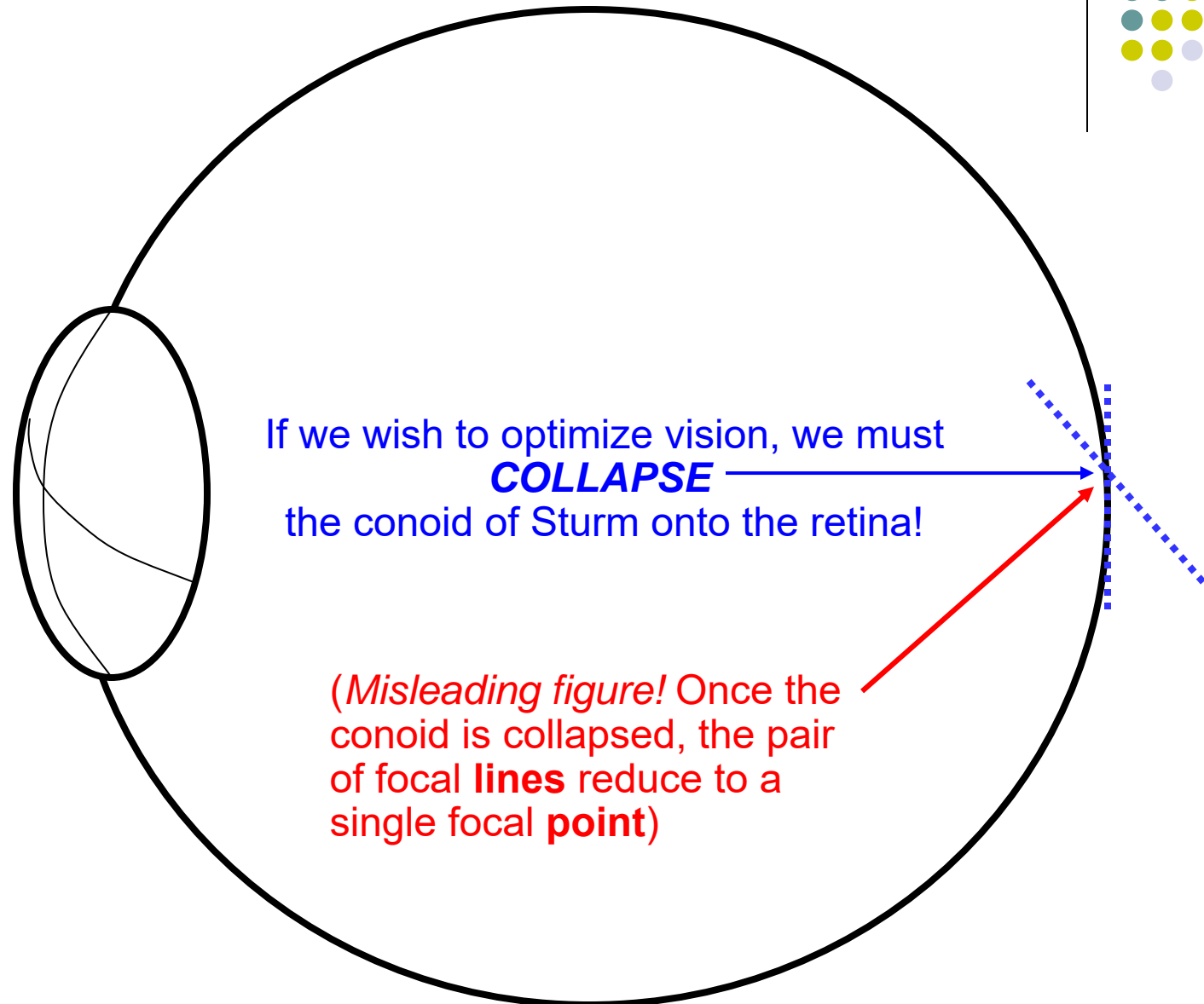
# Astigmatic Eye Error

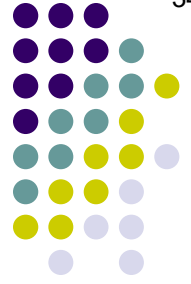




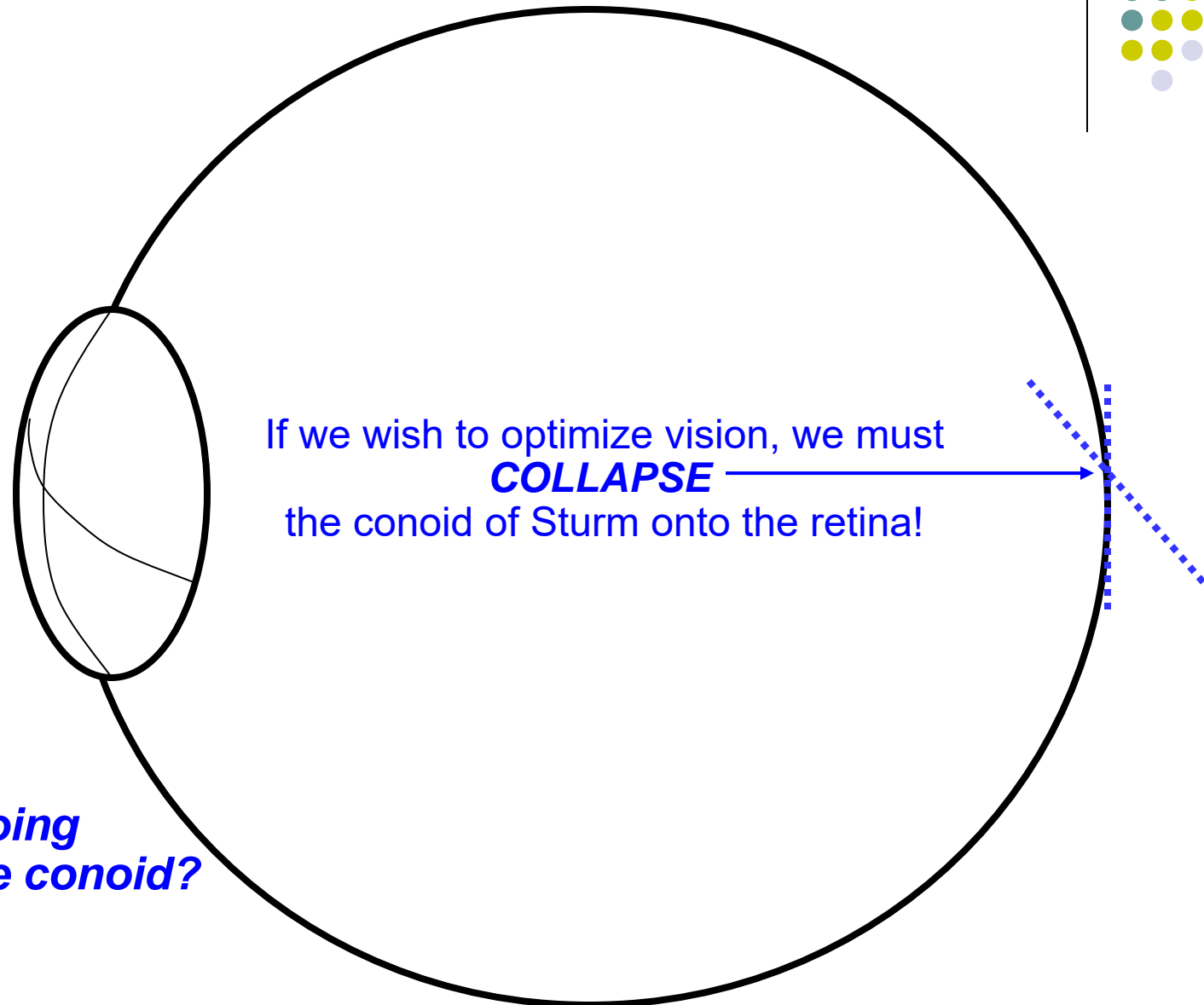


# Astigmatic Eye Error





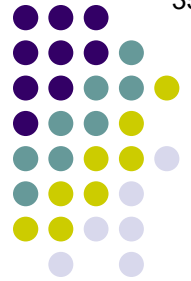
# Astigmatic Eye Error



If we wish to optimize vision, we must  
**COLLAPSE** →  
the conoid of Sturm onto the retina!

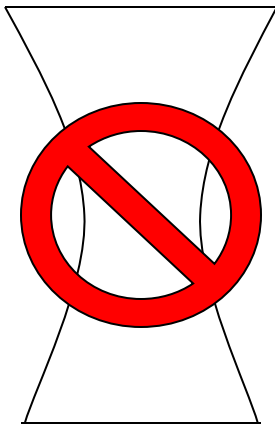
*How are we going  
to collapse the conoid?*

# Astigmatic Eye Error



Eye error S.E. = +1.50

-1.50D lens



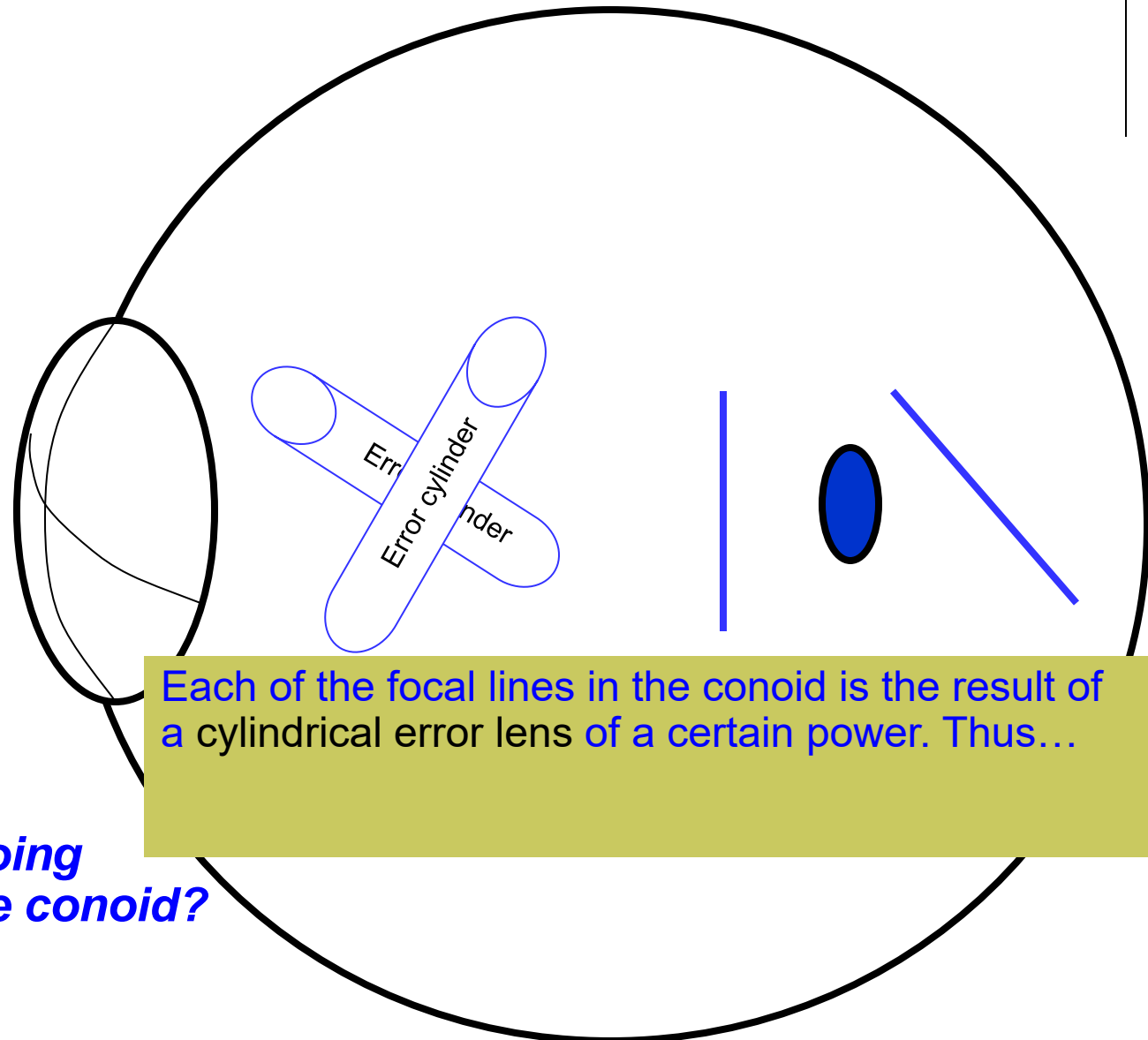
If we wish to optimize vision, we must  
**COLLAPSE** →  
the conoid of Sturm onto the retina!

*How are we going  
to collapse the conoid?*

Not with a **spherical** lens—the best it could do would be to put the CoLC on the retina, with suboptimal resulting vision

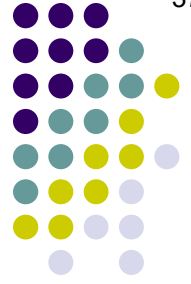


# Astigmatic Eye Error

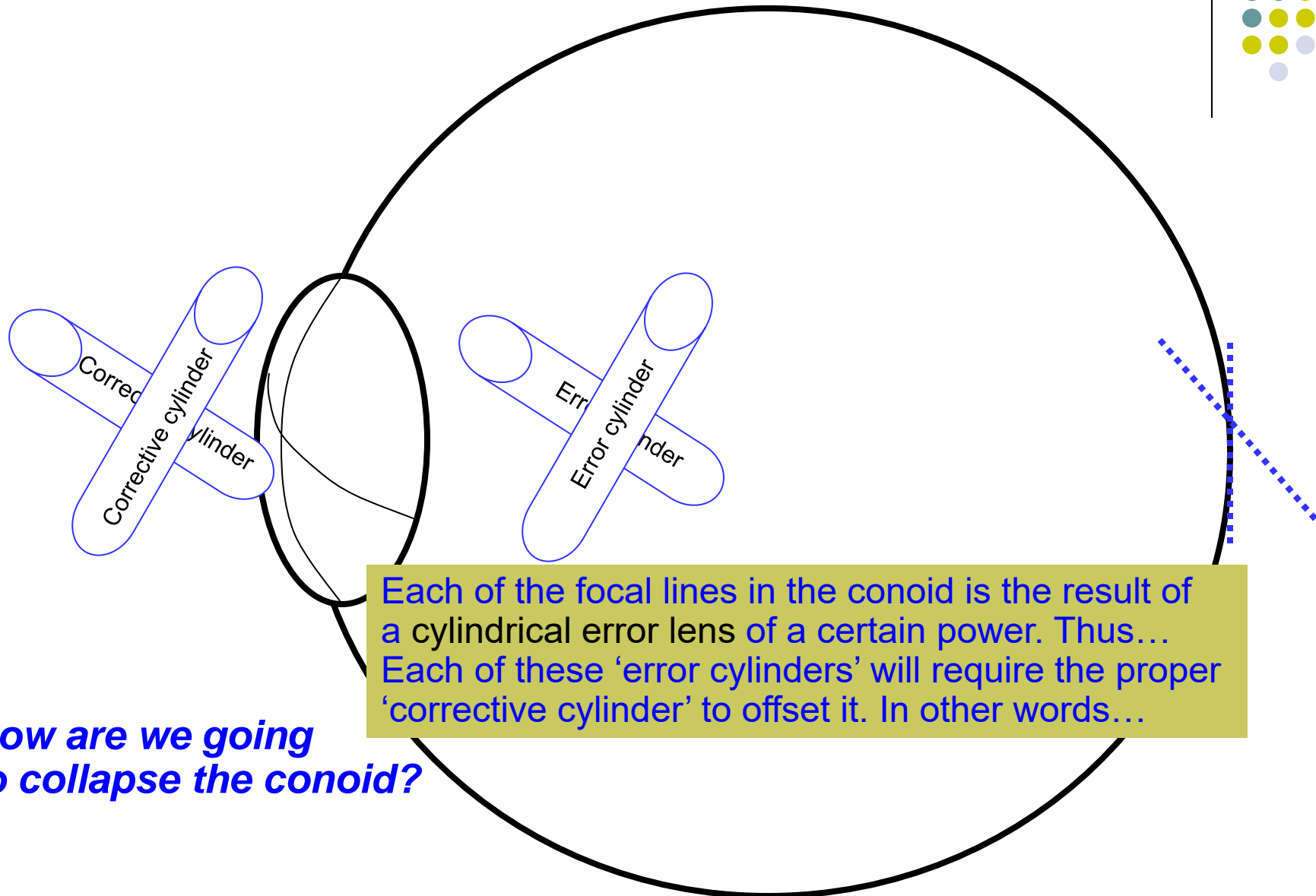


Each of the focal lines in the conoid is the result of a cylindrical error lens of a certain power. Thus...

*How are we going to collapse the conoid?*

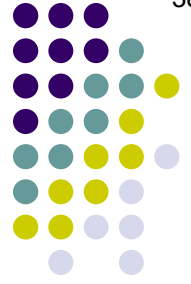


# Astigmatic Eye Error



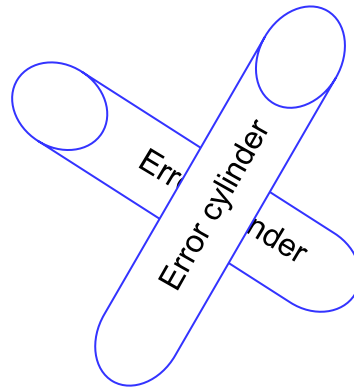
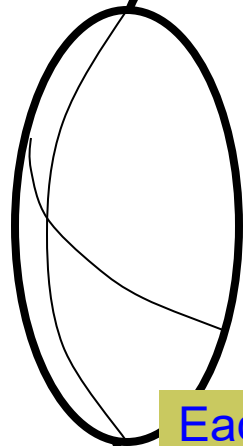
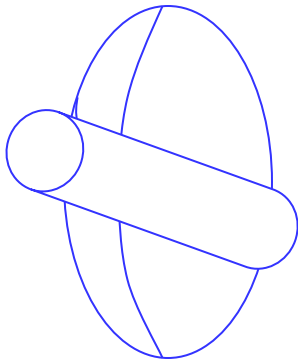
***How are we going to collapse the conoid?***

Each of the focal lines in the conoid is the result of a cylindrical error lens of a certain power. Thus... Each of these 'error cylinders' will require the proper 'corrective cylinder' to offset it. In other words...



# Astigmatic Eye Error

We must use a combo of cylinders--i.e., a ***SPHEROCYLINDRICAL LENS***--to collapse the conoid of Sturm and thereby fully correct an astigmatic refractive error!

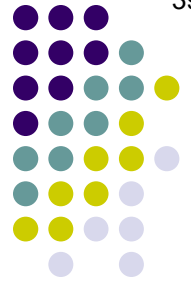
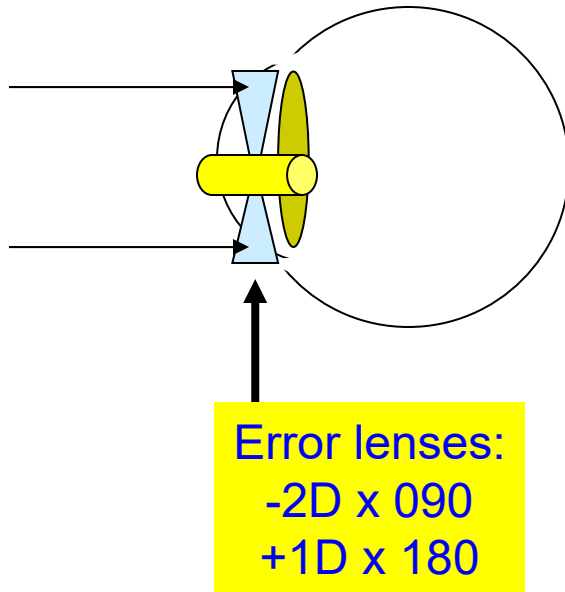


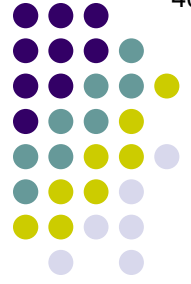
Each of the focal lines in the conoid is the result of a cylindrical error lens of a certain power. Thus... Each of these 'error cylinders' will require the proper 'corrective cylinder' to offset it. In other words...

***How are we going to collapse the conoid?***

# Astigmatic Eye Error

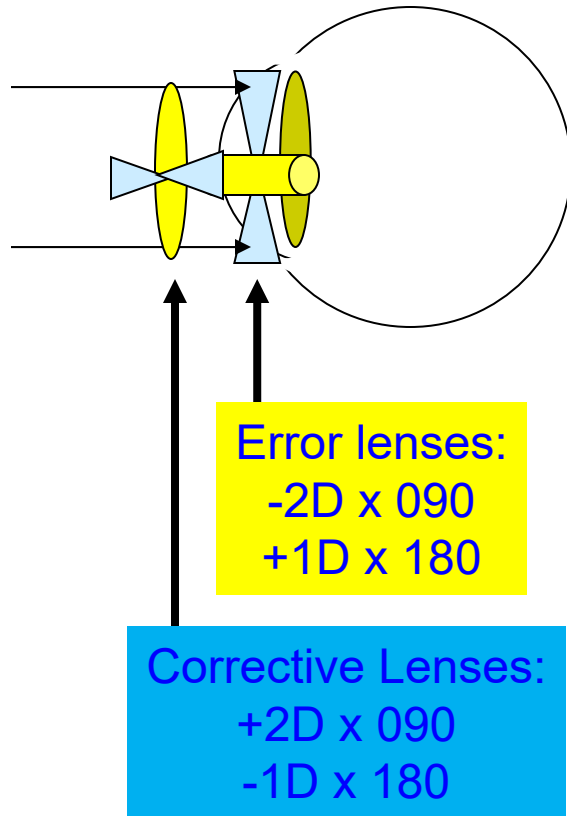
*For example...*





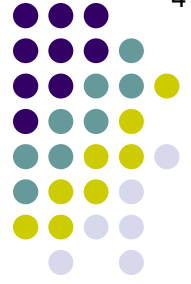
# Astigmatic Eye Error

*For example...*



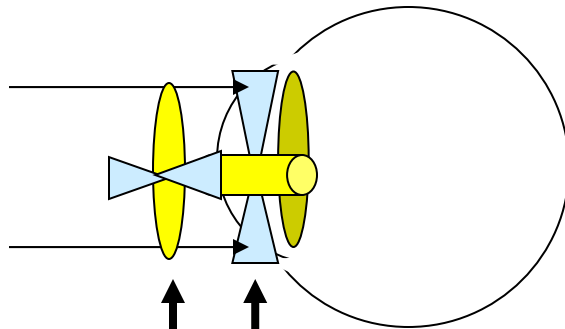
*(Note: These corrective lenses do not account for vertex distance)*





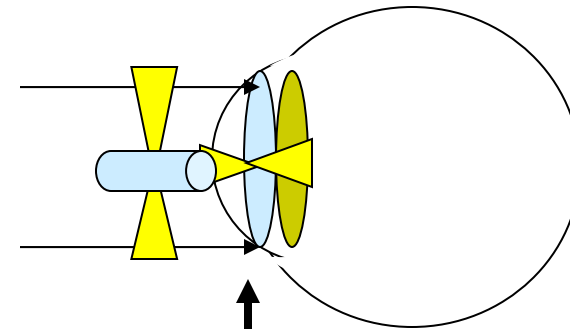
# Astigmatic Eye Error

*For example...*



Error lenses:  
 $-2\text{D} \times 090$   
 $+1\text{D} \times 180$

Corrective Lenses:  
 $+2\text{D} \times 090$   
 $-1\text{D} \times 180$



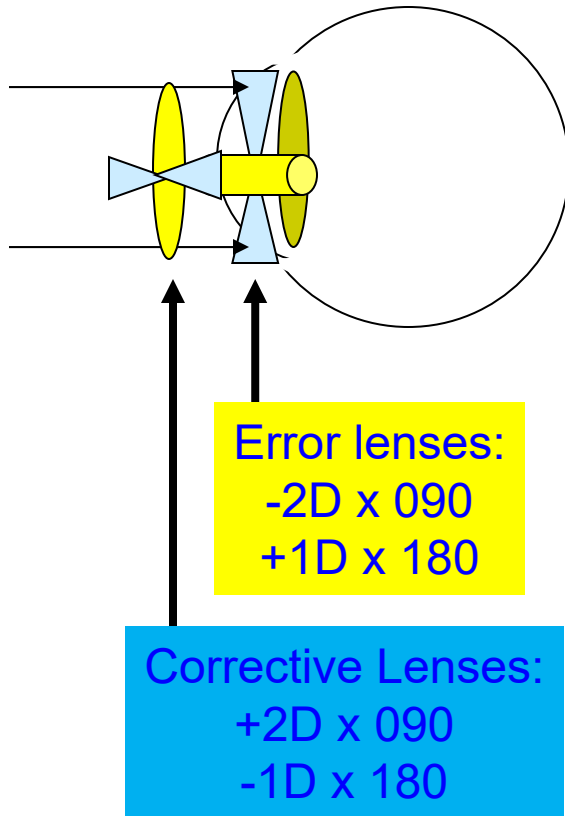
Error Lenses:  
 $+4 \times 090$   
 $-1.5 \times 180$

*(Note: These corrective lenses do not account for vertex distance)*

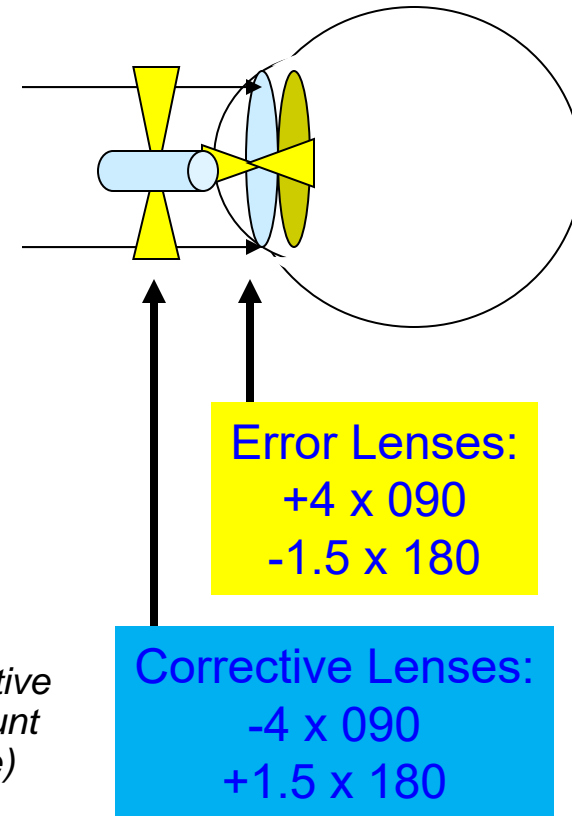


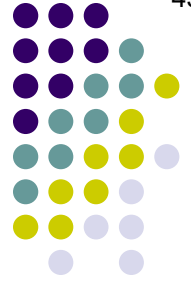
# Astigmatic Eye Error

*For example...*



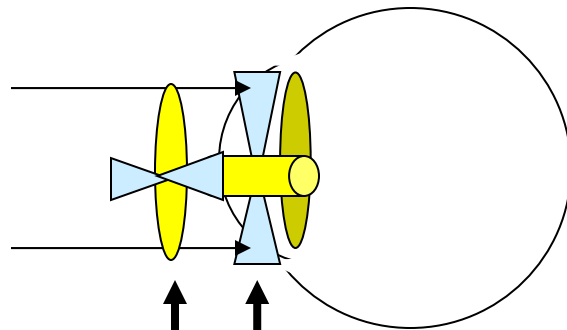
*(Note: These corrective lenses do not account for vertex distance)*





# Astigmatic Eye Error

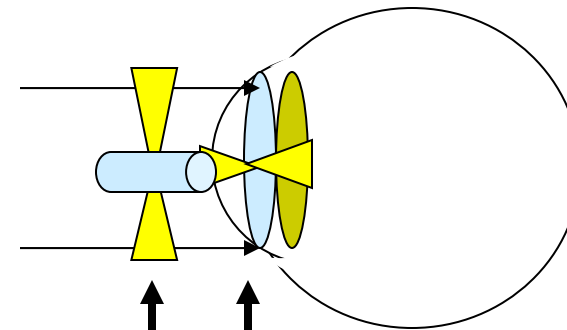
*For example...*



Error lenses:  
 $-2D \times 090$   
 $+1D \times 180$

Corrective Lenses:  
 $+2D \times 090$   
 $-1D \times 180$

*(Note: These corrective lenses do not account for vertex distance)*



Error Lenses:  
 $+4 \times 090$   
 $-1.5 \times 180$

Corrective Lenses:  
 $-4 \times 090$   
 $+1.5 \times 180$

*We will take up the topic of how one refracts an astigmatic eye in the next chapter*