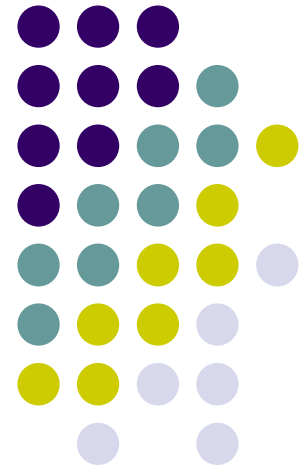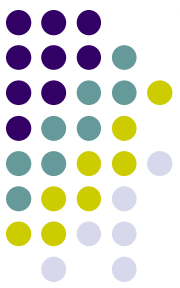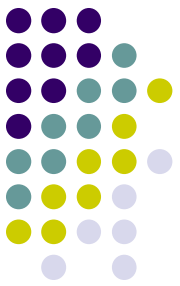# Angular Magnification

Basic Optics, Chapter 21
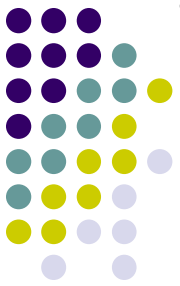
Transverse Magnification

- But first, let's recall some of the facts about *transverse* magnification…
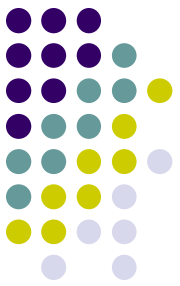
Transverse Magnification

- But first, let's recall some of the facts about *transverse* magnification…
  - Transverse magnification refers to the actual sizes of images and objects, **not** to how they appear to an observer
    - How big they *are*, not how big they *look*

Transverse Magnification

But what about objects and images located at *infinity?*

Transverse Magnification

Transverse mag = $\dfrac{\textit{Image height}}{\textit{Object height}}$

= $\dfrac{\textit{Image distance (v)}}{\textit{Object distance (u)}}$

But what about objects and images located at ***infinity?***

Transverse Magnification

Transverse mag = $\dfrac{\textit{Image height}}{\textit{Object height}}$ **=** *?*

= $\dfrac{\textit{Image distance (v)}}{\textit{Object distance (u)}}$

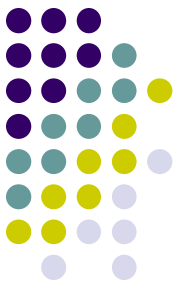So, if the object is at infinity, then the transverse mag is *undefined* mathematically, approaching *zero. (Huh?)*

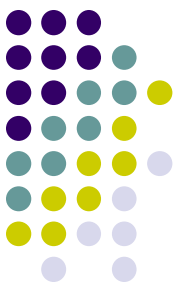But what about objects and images located at *infinity?*

Transverse Magnification

Transverse mag = $\dfrac{\textit{Image height}}{\textit{Object height}}$ **= ?**

$= \dfrac{\textit{Image distance (v)}}{\textit{Object distance (u)}}$

Likewise, if the image is at infinity, then the transverse mag is *infinitely large. (Huh?)*

So, if the object is at infinity, then the transverse mag is *undefined* mathematically, approaching *zero. (Huh?)*
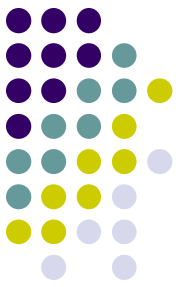
But what about objects and images located at *infinity?*

Transverse Magnification

- In addition, consider the optics of an *afocal system* (e.g., a telescope)

Low plus lens   High plus lens

Parallel rays *from* an object at infinity

Parallel rays *to* an image at infinity

Astronomical (Keplerian) telescope

Low plus lens
High minus lens

Parallel rays *from* an object at infinity

Parallel rays *to* an image at infinity

Galilean (terrestrial) telescope

Transverse Magnification

- In addition, consider the optics of an *afocal system* (e.g., a telescope)

Low plus lens High plus lens

Parallel rays
from an object
at infinity

Parallel rays
to an image
at infinity

Astronomical (Keplerian) telescope

Low plus lens
High minus lens
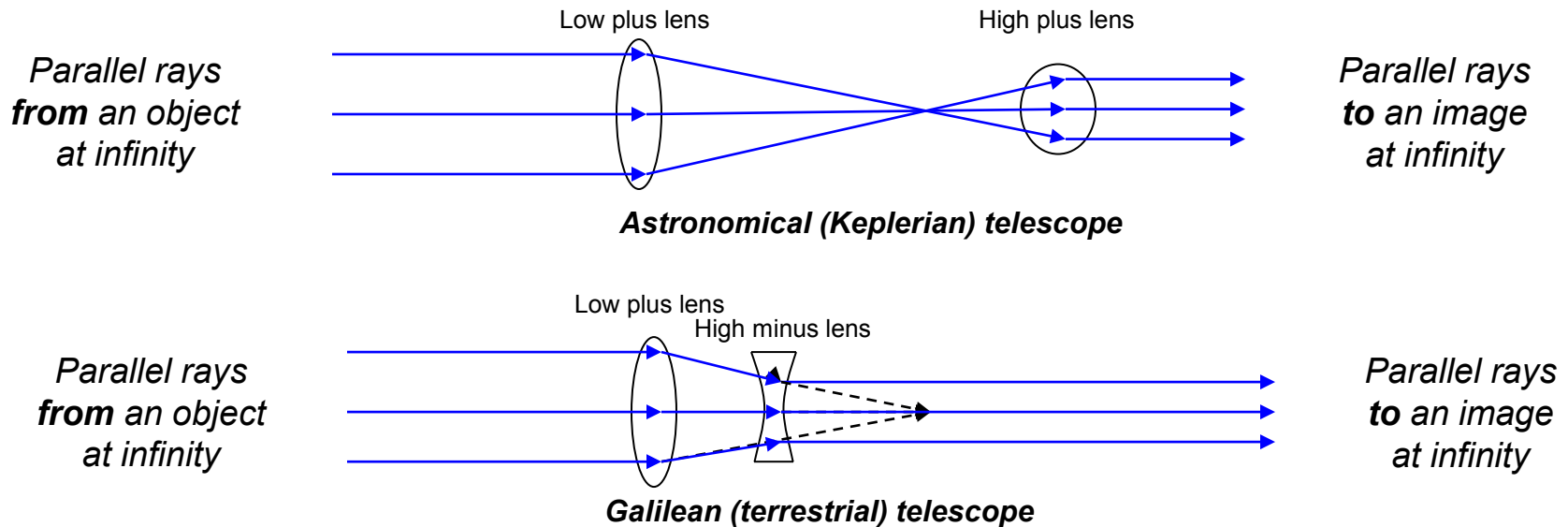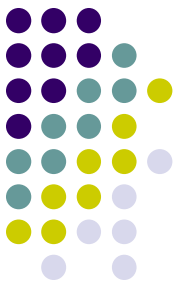
Parallel rays
from an object
at infinity

Parallel rays
to an image
at infinity

Galilean (terrestrial) telescope
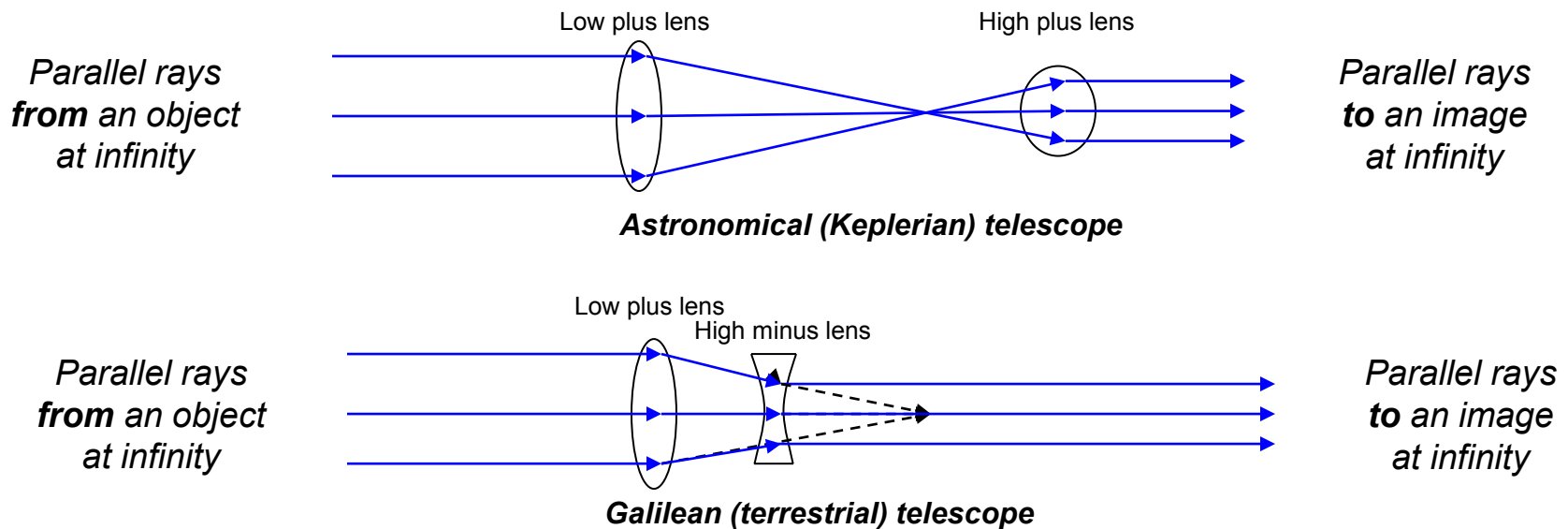
Note that focused rays are not involved—that is, telescopes have parallel (i.e., afocal) rays both coming in and going out

Transverse Magnification

- In addition, consider the optics of an *afocal system* (e.g., a telescope)

Low plus lens   High plus lens

Parallel rays
from *an object*
at infinity

Parallel rays
to *an image*
at infinity

Astronomical (Keplerian) telescope

Low plus lens
High minus lens

Parallel rays
from *an object*
at infinity

Parallel rays
to *an image*
at infinity

Galilean (terrestrial) telescope
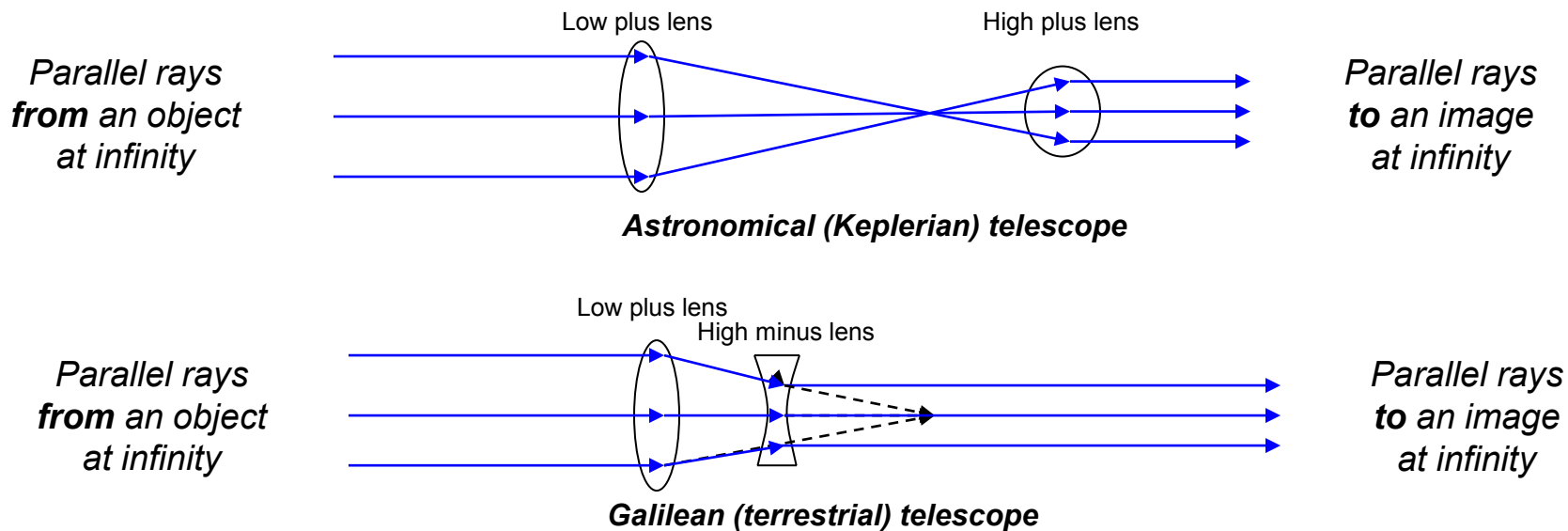
So, for telescopes, the transverse magnification would seem to be:

$$\frac{\text{Image distance } (v)}{\text{Object distance } (u)} = \infty / \infty = 1.0$$

In other words, no magnification at all!

Transverse Magnification

- In addition, consider the optics of an *afocal system* (e.g., a telescope)

Low plus lens   High plus lens
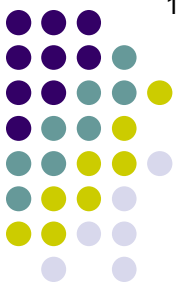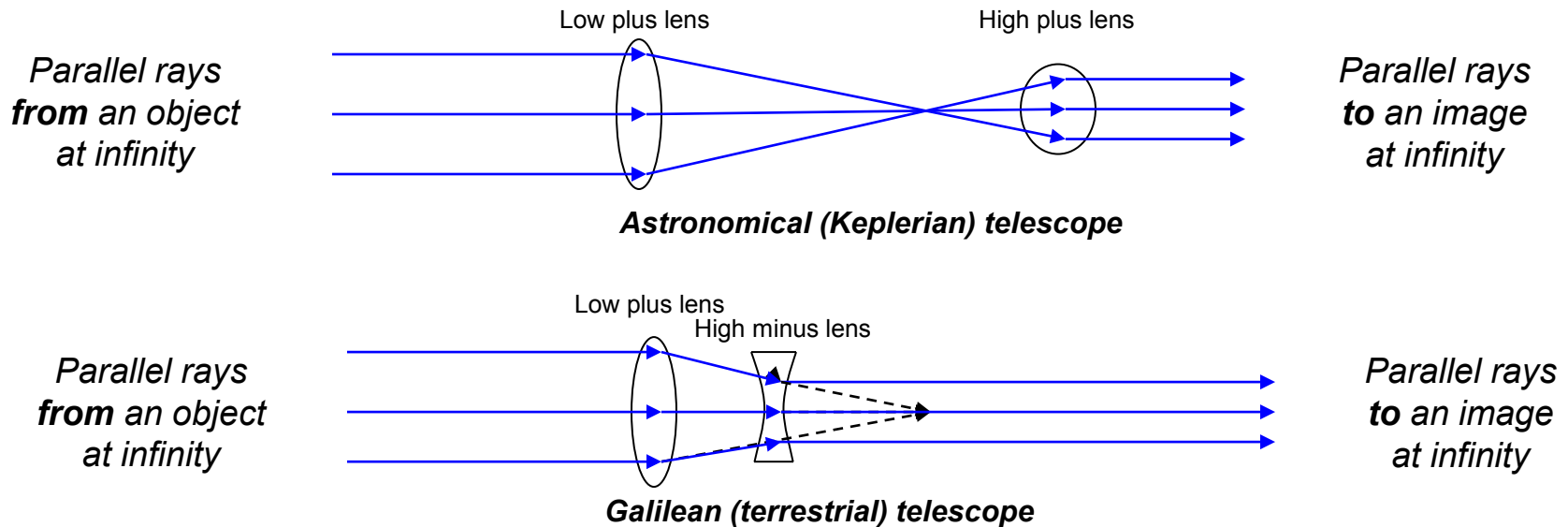
Parallel rays from an object at infinity

Parallel rays to an image at infinity

Astronomical (Keplerian) telescope

Low plus lens
High minus lens

Parallel rays from an object at infinity

Parallel rays to an image at infinity

Galilean (terrestrial) telescope

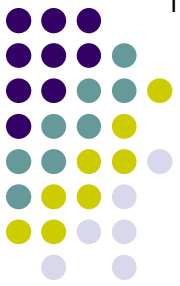So, for telescopes, the transverse magnification would seem to be:

$$\frac{\text{Image distance } (v)}{\text{Object distance } (u)} = \infty/\infty = 1.0$$

What good is a telescope that doesn't magnify?

Transverse Magnification

- Clearly, transverse mag cannot meet all our 'magnification needs'

- We also need a measure that addresses the **apparent** sizes of objects and images, not just **actual** sizes
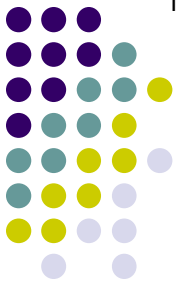
Angular **Magnification**

- Clearly, transverse mag cannot meet all our 'magnification needs'

- We also need a measure that addresses the **apparent** sizes of objects and images, not just **actual** sizes
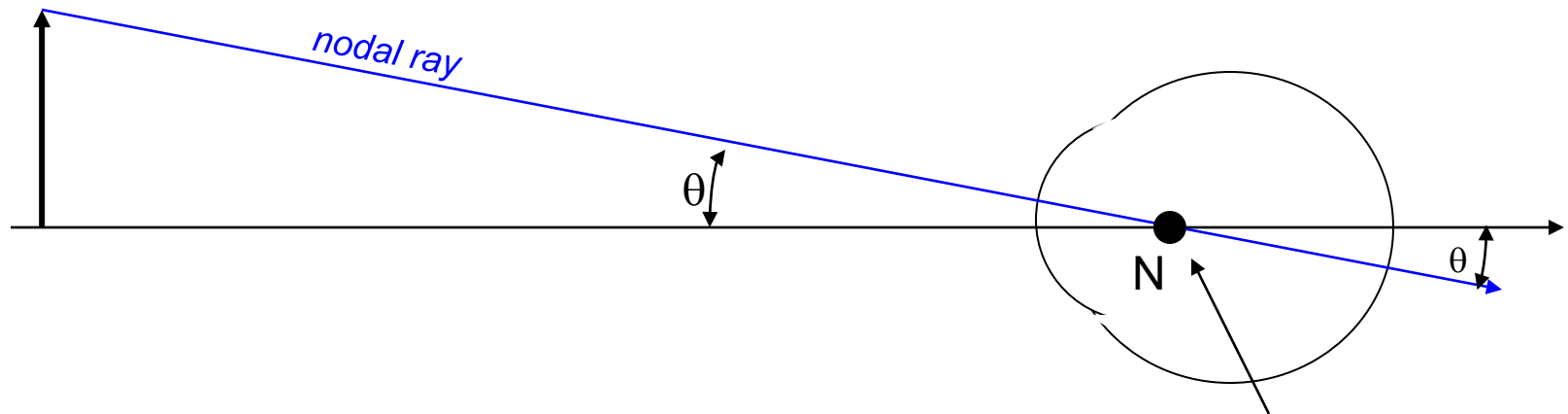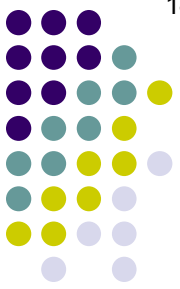
- That measure is ***angular magnification***

  - How big objects *look*, not how big they *are*

Angular Magnification
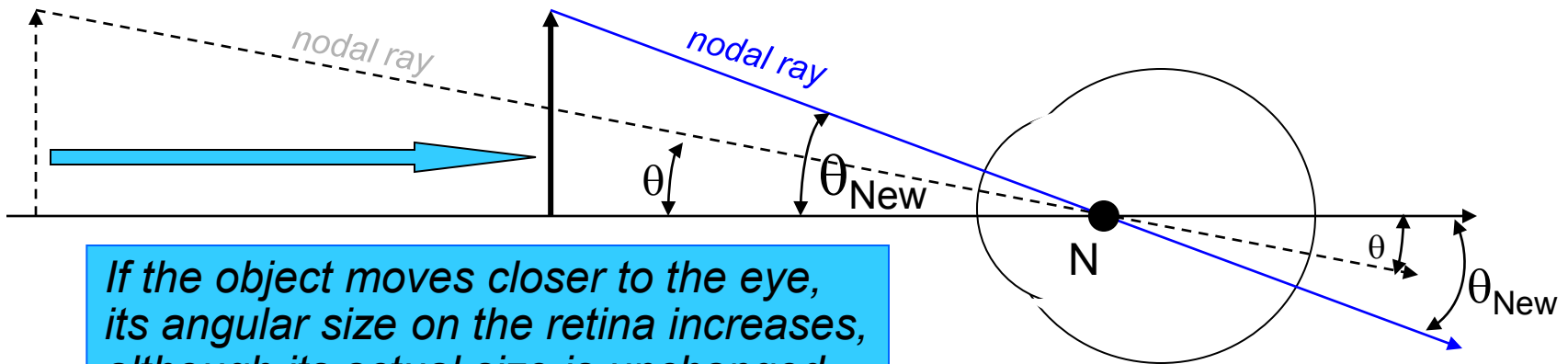
- *Angular size* is determined by the angular extent of retina an image subtenses (θ)

nodal ray

θ

N

θ

Just as in any other optical system, the *nodal point of the eye* determines image location when ray tracing

Angular Magnification

- *Angular size* is determined by the angular extent of retina an image subtenses (θ)

nodal ray

nodal ray

θ

θ$_{New}$

N

θ

θ$_{New}$
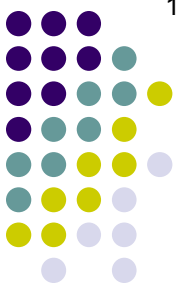
If the object moves closer to the eye, its angular size on the retina increases, although its actual size is unchanged

Angular Magnification

- *Angular size* is determined by the angular extent of retina an image subtenses (θ)

nodal ray

N

θ
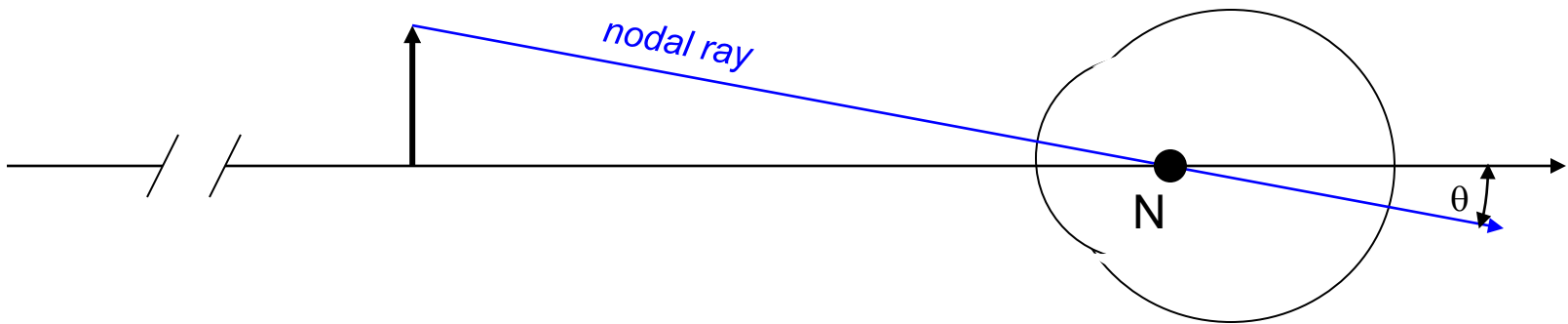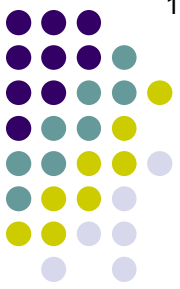
Consider this optical system…

Angular Magnification

- *Angular size* is determined by the angular extent of retina an image subtenses (θ)

Plus lens

nodal ray

F_1

N

N

θ

Primary focal plane of plus lens

Consider this optical system…What would happen if we inserted a plus lens such that the object was located at its primary focal plane?

Angular Magnification

- *Angular size* is determined by the angular extent of retina an image subtenses (θ)

All the rays will leave the lens parallel to the nodal ray, and…

Plus lens

nodal ray

nodal ray (lens)

F_1

N

N

θ

Primary focal plane of plus lens

Consider this optical system…What would happen if we inserted a plus lens such that the object was located at its primary focal plane?

Angular Magnification

- *Angular size* is determined by the angular extent of retina an image subtenses (θ)

All the rays will leave the lens parallel to the nodal ray, and…

One of those rays will pass through the nodal point of the eye, subtensing a new angular size

Plus lens

nodal ray

nodal ray (lens)

nodal ray (eye)

F_1

N

N

θ

θ'

Primary focal plane of plus lens

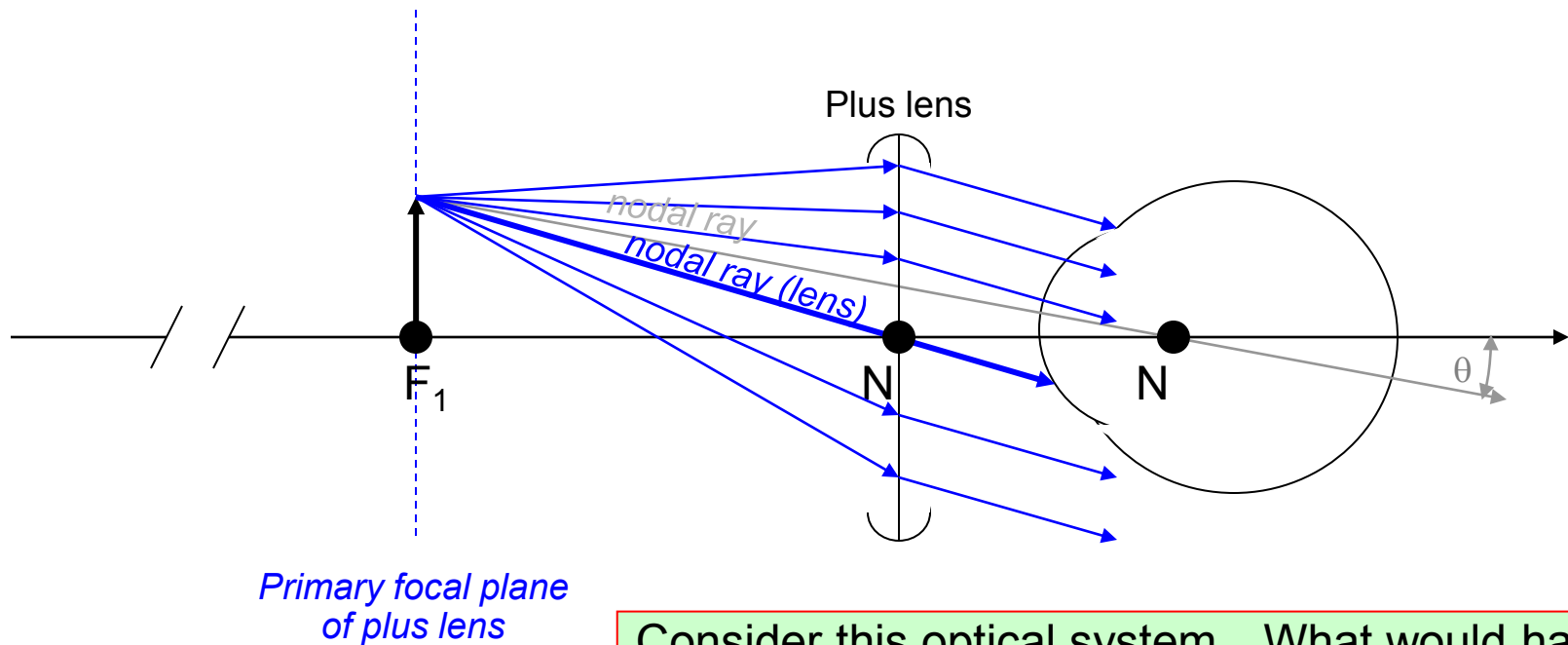Consider this optical system…What would happen if we inserted a plus lens such that the object was located at its primary focal plane?

Angular Magnification

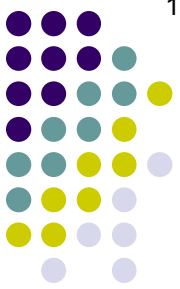- *Angular size* is determined by the angular extent of retina an image subtenses (θ)

We can trace this ray back and see where a magnified virtual image is located

All the rays will leave the lens parallel to the nodal ray, and…

One of those rays will pass through the nodal point of the eye, subtensing a new angular size

Virtual image at infinity

It appears bigger!

Plus lens

nodal ray

nodal ray (lens)

nodal ray (eye)

F_1

N

N

θ

θ'

Primary focal plane of plus lens

Consider this optical system…What would happen if we inserted a plus lens such that the object was located at its primary focal plane?
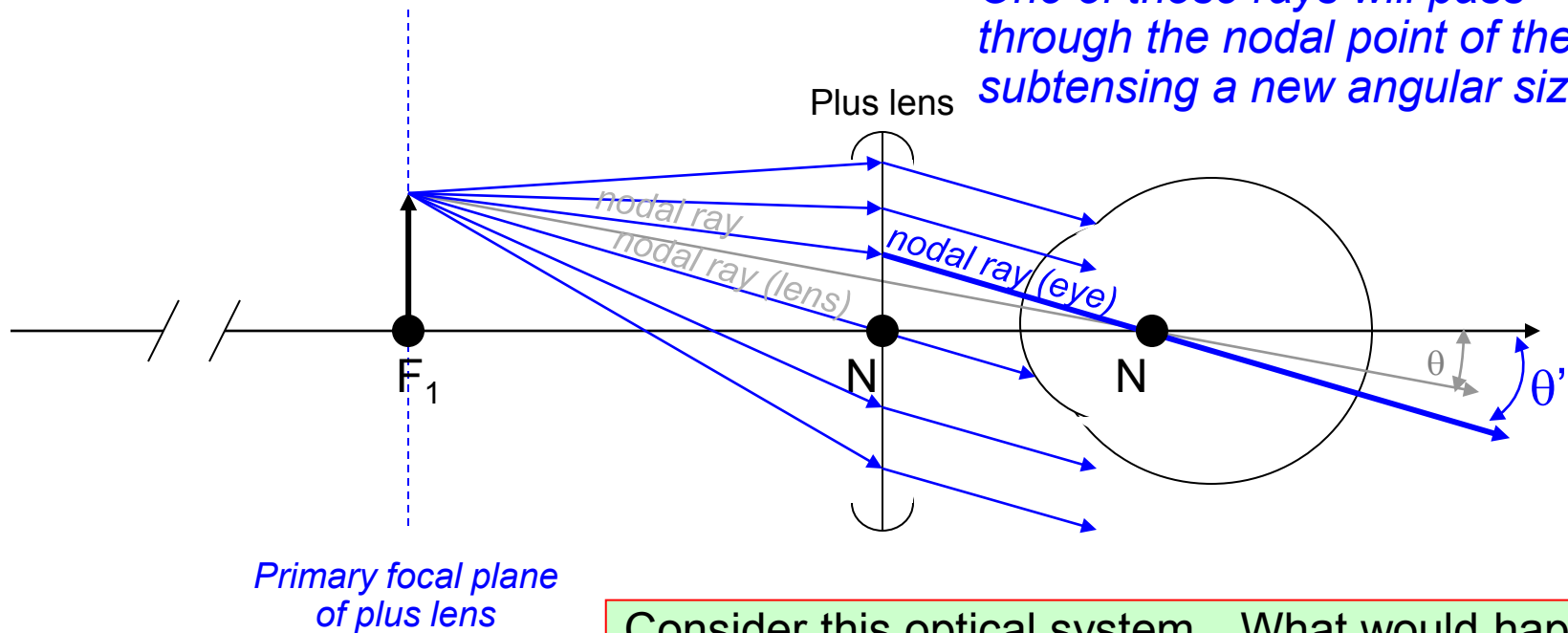
Angular Magnification

- *Angular size* is determined by the angular extent of retina an image subtenses (θ)

We can trace this ray back and see where a magnified virtual image is located
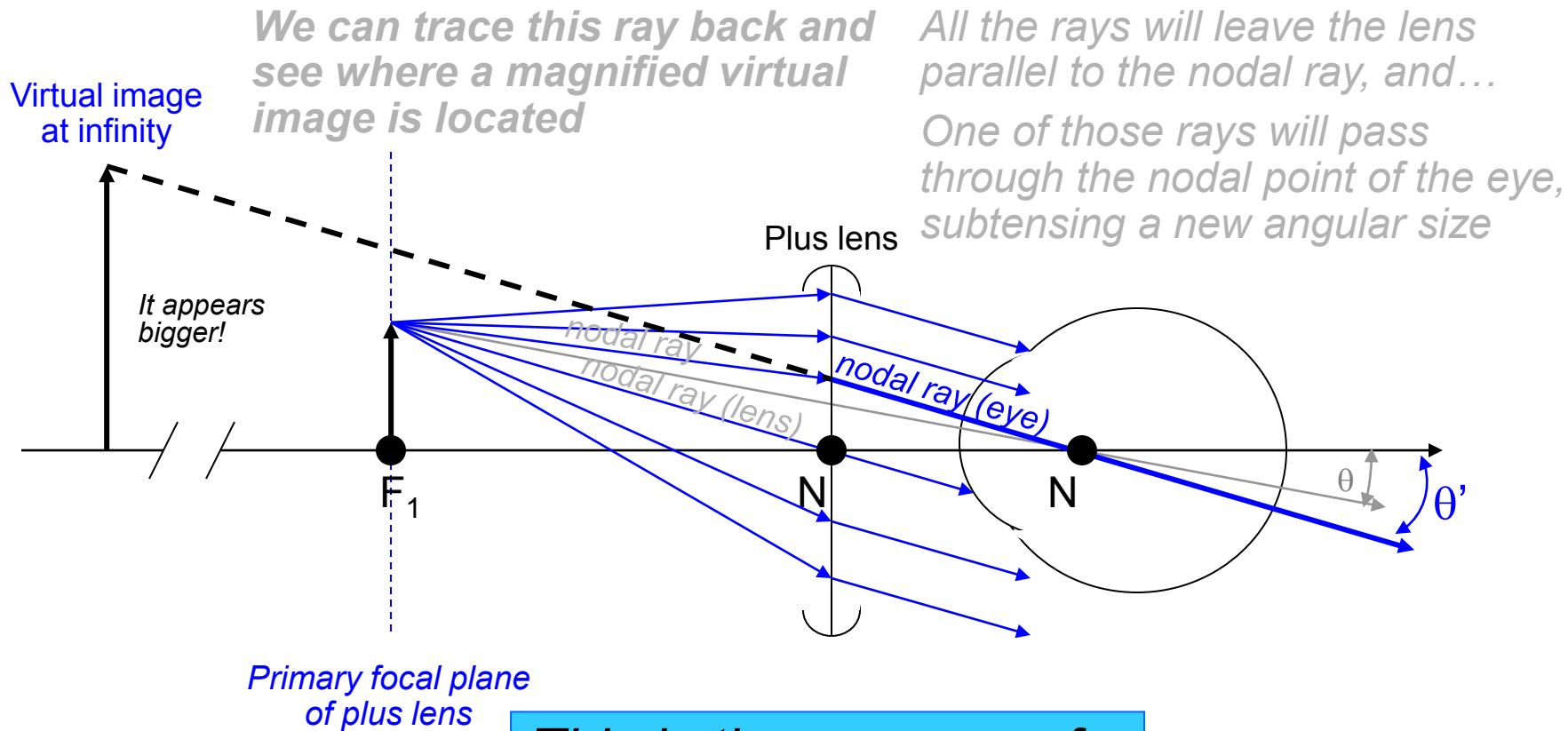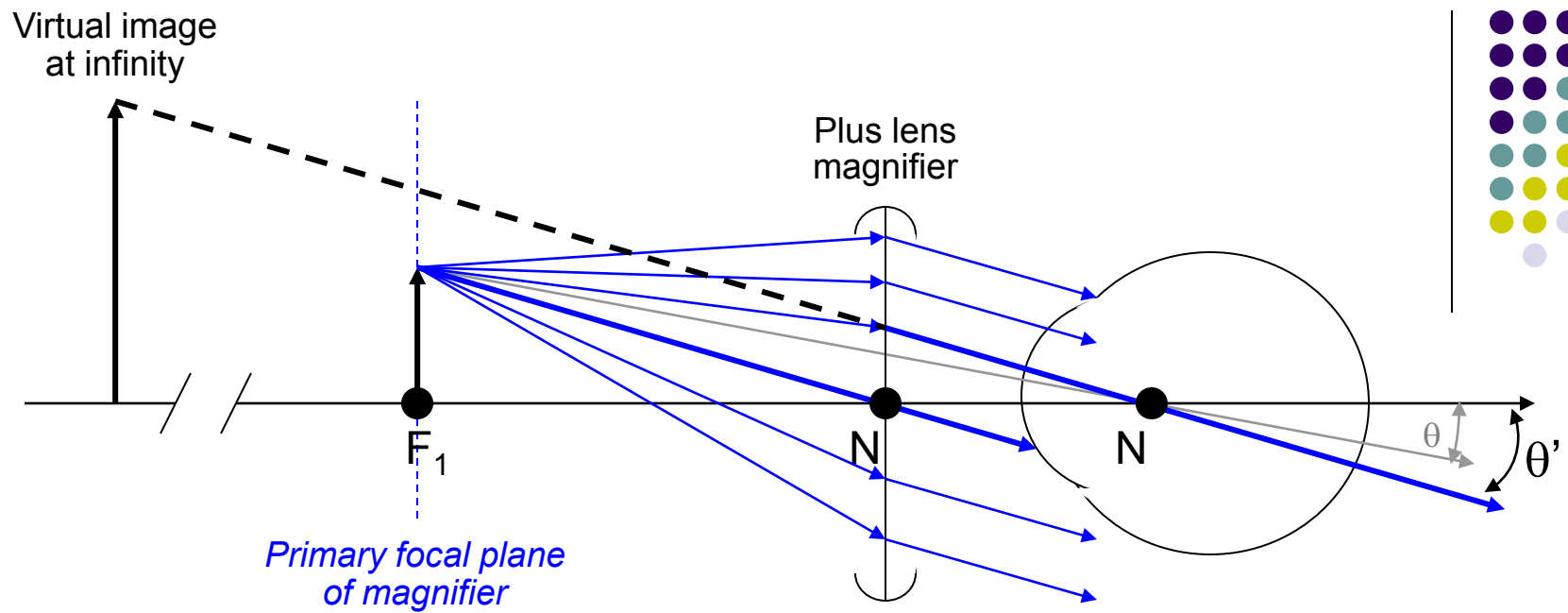
All the rays will leave the lens parallel to the nodal ray, and…

One of those rays will pass through the nodal point of the eye, subtensing a new angular size

Virtual image at infinity

It appears bigger!

Plus lens

nodal ray

nodal ray (lens)

nodal ray (eye)

F_1

N

N

θ

θ'

Primary focal plane of plus lens

This is the essence of a ***simple magnifier***

Virtual image
at infinity

Plus lens
magnifier

F_1

N

N

θ

θ'

Primary focal plane
of magnifier

An increase in the dioptric
power of the plus lens…

***Higher* power**
plus lens
magnifier

F_1

N

N

θ

θ'

Primary focal plane
of magnifier

Virtual image
at infinity

Plus lens
magnifier

F₁

N

N

θ

θ'
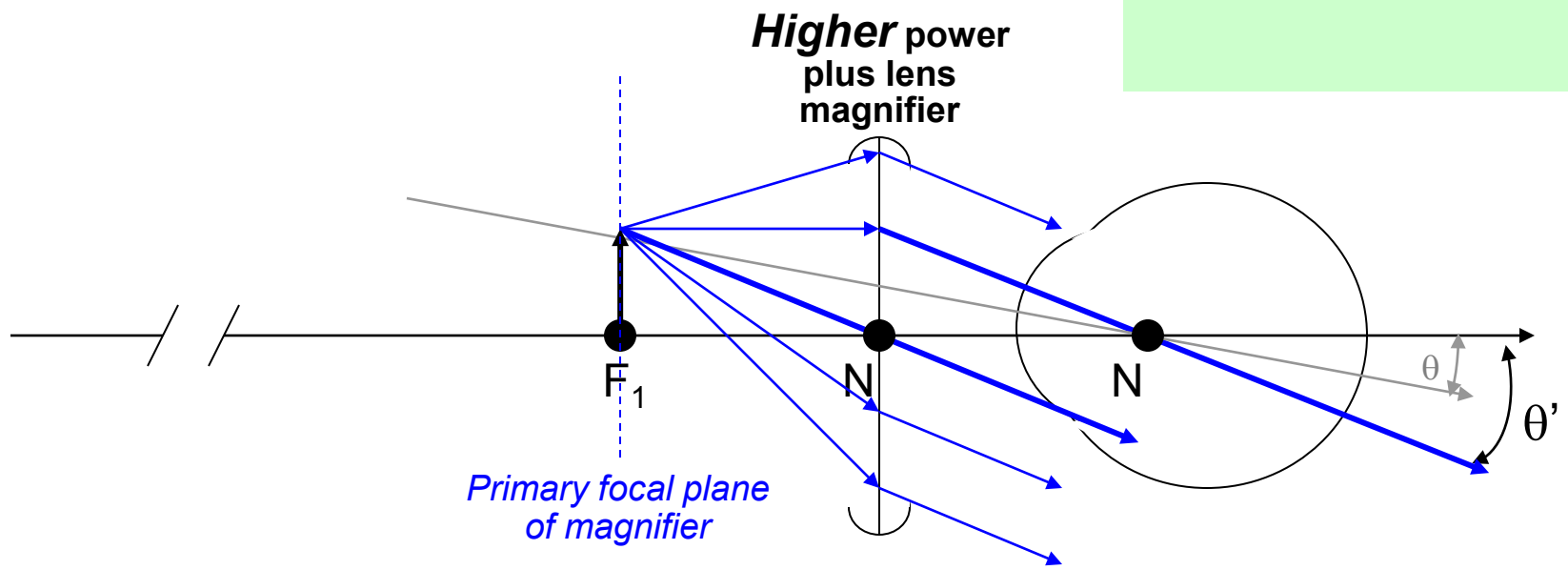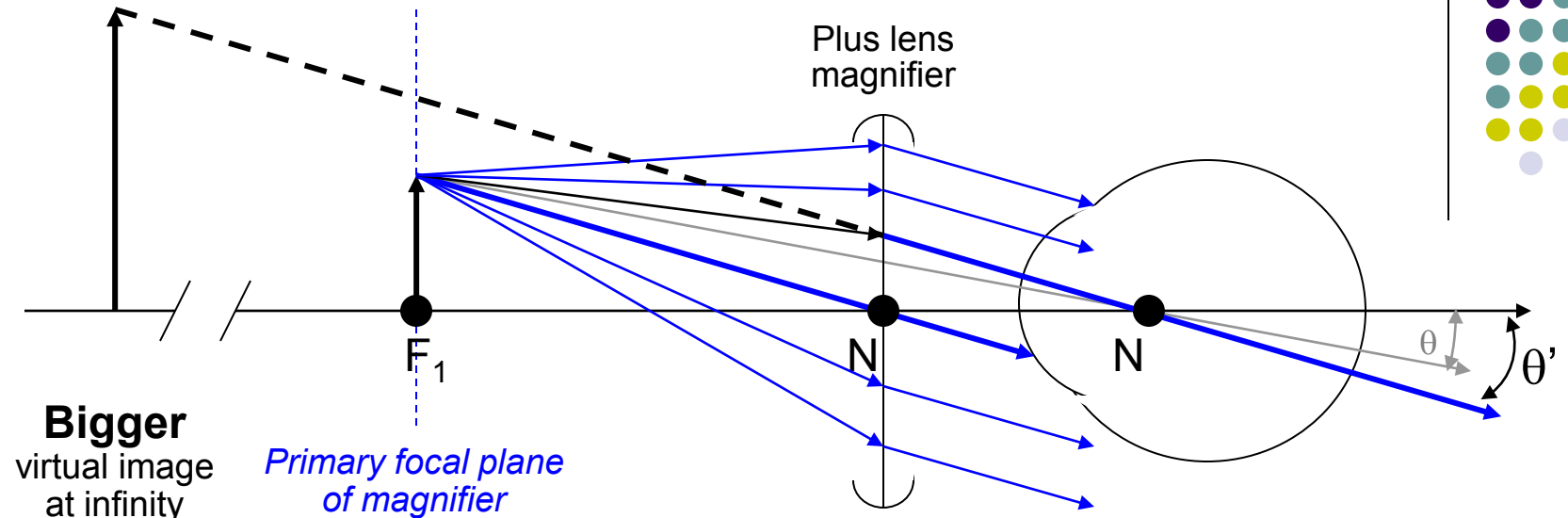
Bigger
virtual image
at infinity

*Primary focal plane
of magnifier*
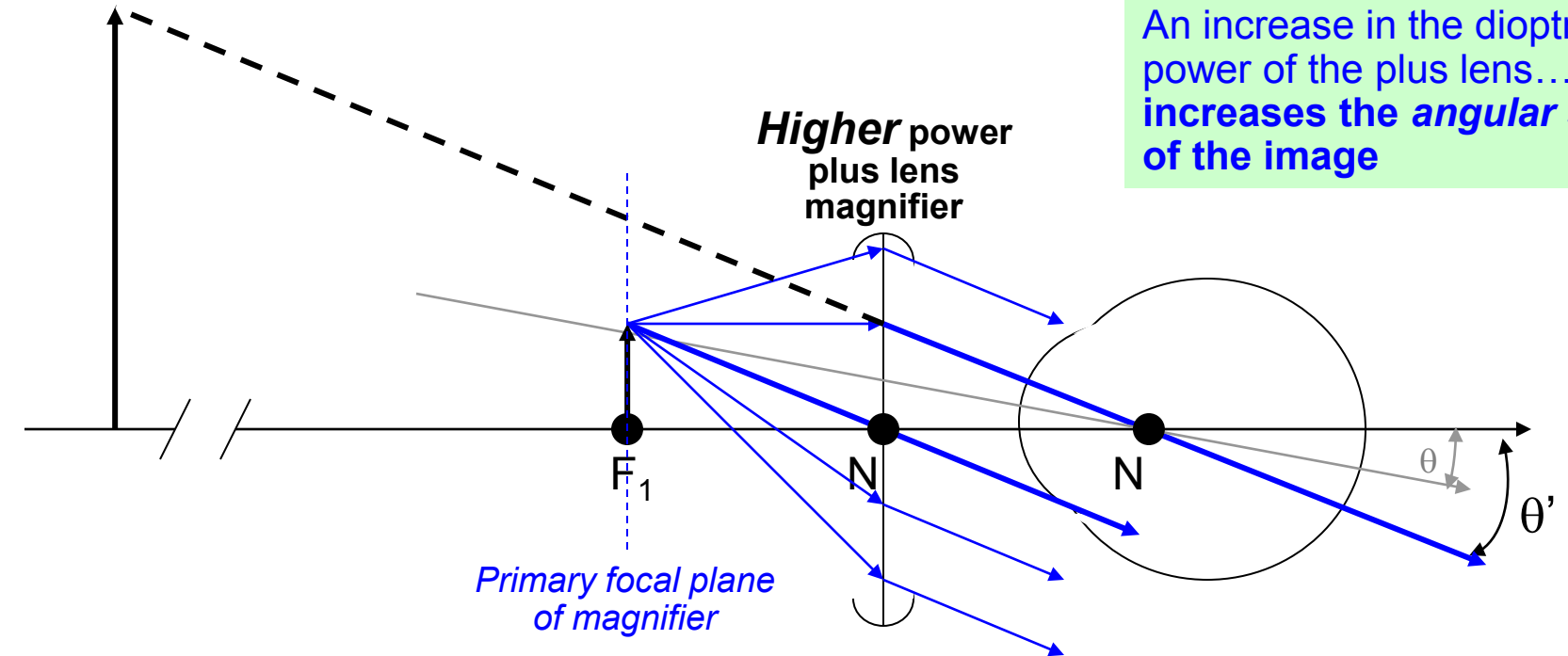
Higher **power
plus lens
magnifier**

An increase in the dioptric
power of the plus lens…
**increases the *angular size*
of the image**

F₁

N

N

θ

θ'

*Primary focal plane
of magnifier*

Virtual image
at infinity

Plus lens
magnifier

F₁

N

N

θ

θ'

*Primary focal plane
of magnifier*

Higher **power**
plus lens
magnifier

F₁

N

N

θ

θ'

*Primary focal plane
of magnifier*

An increase in the dioptric
power of the plus lens…
**increases the *angular size*
of the image**

Angular size **(θ') =**
Object size x *lens power*

Angular Magnification

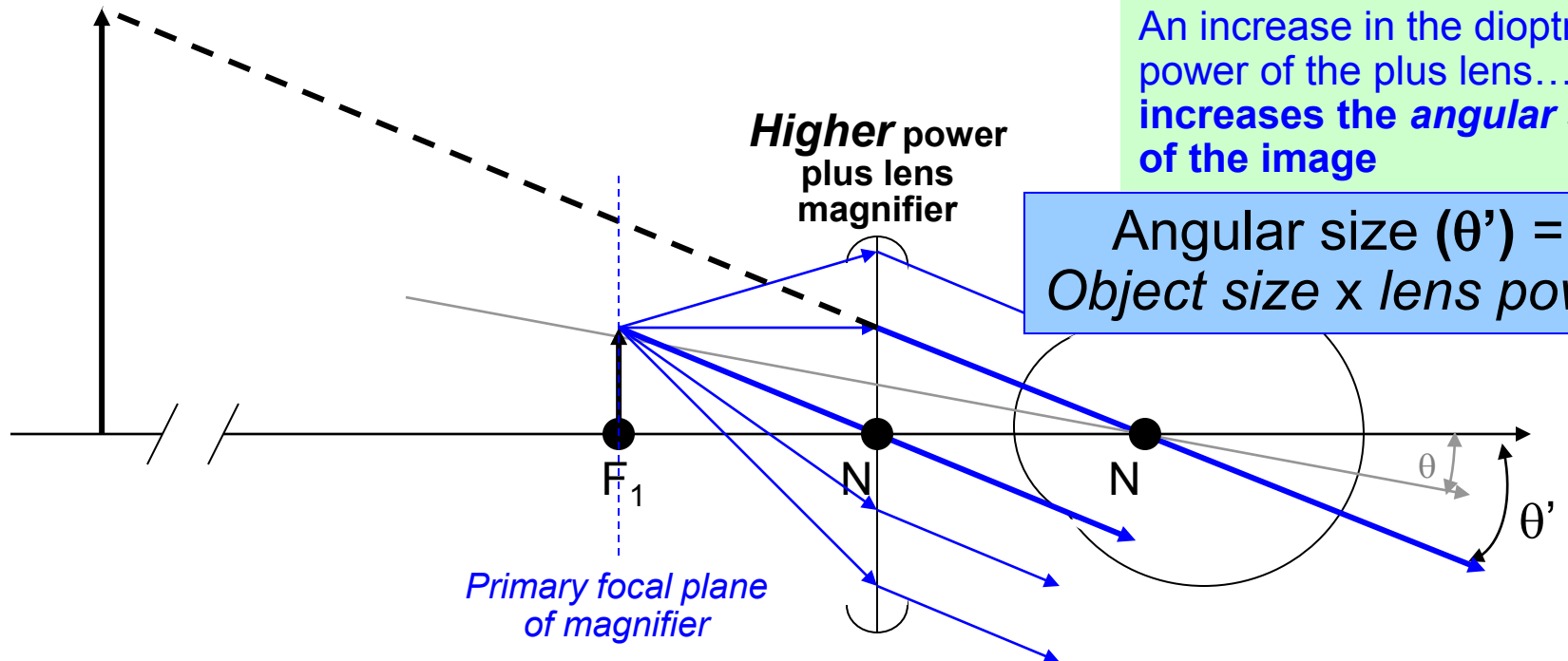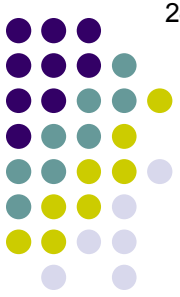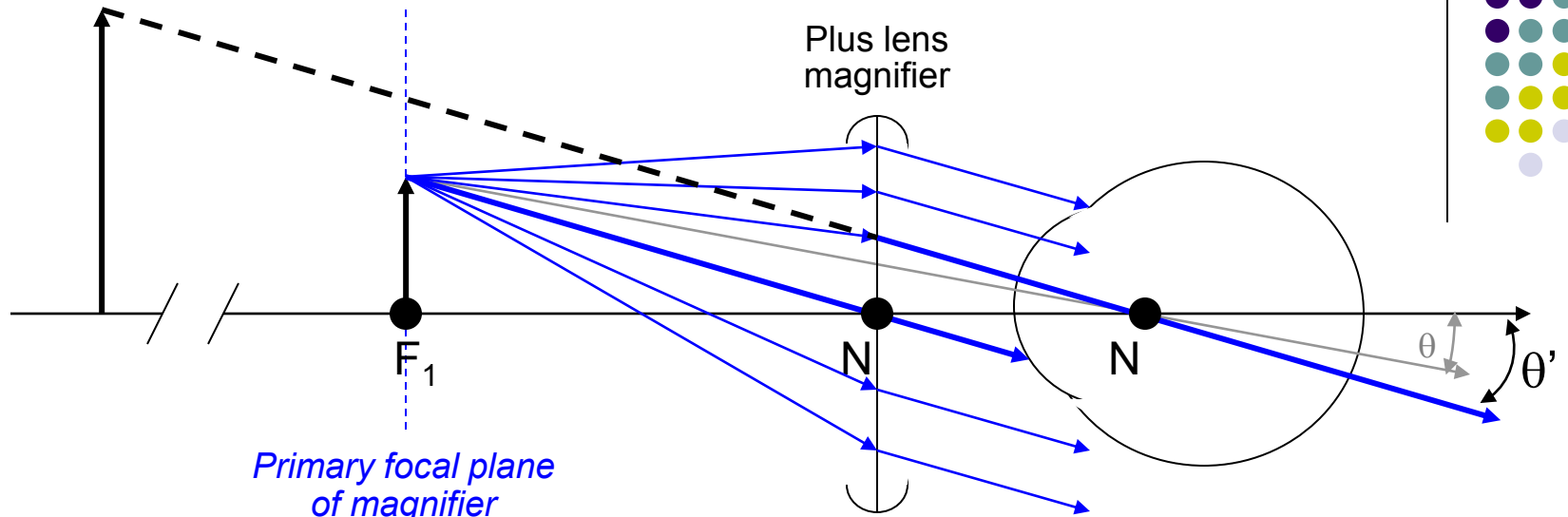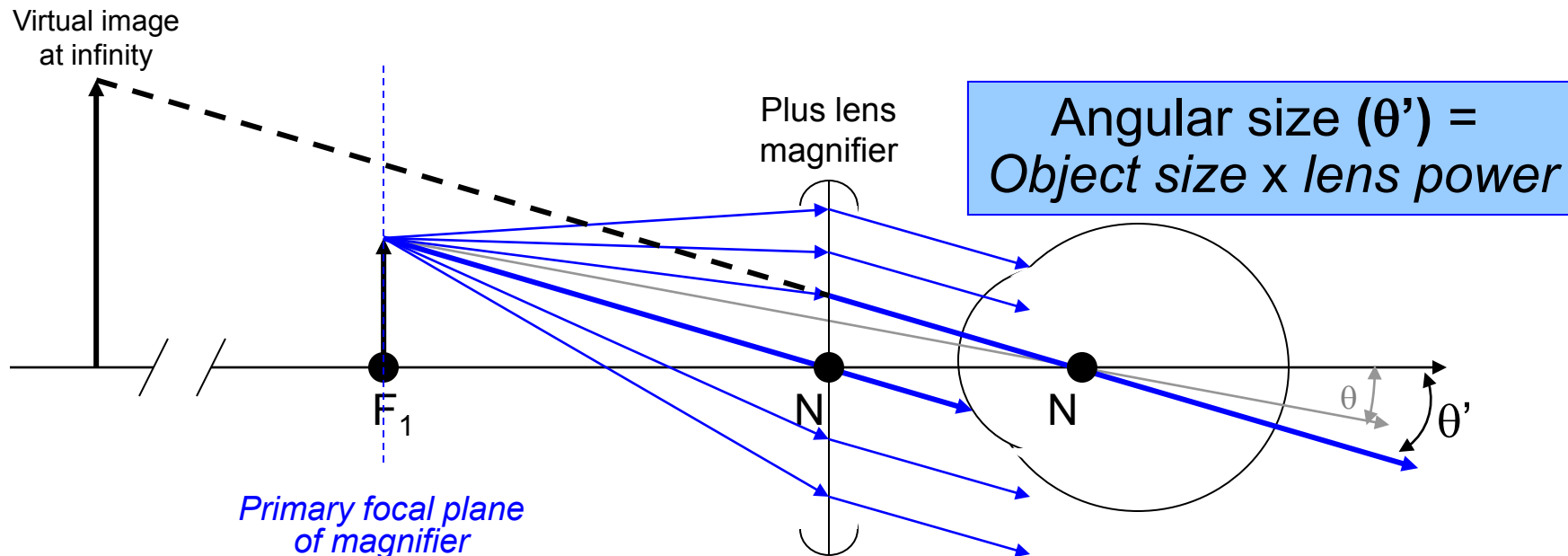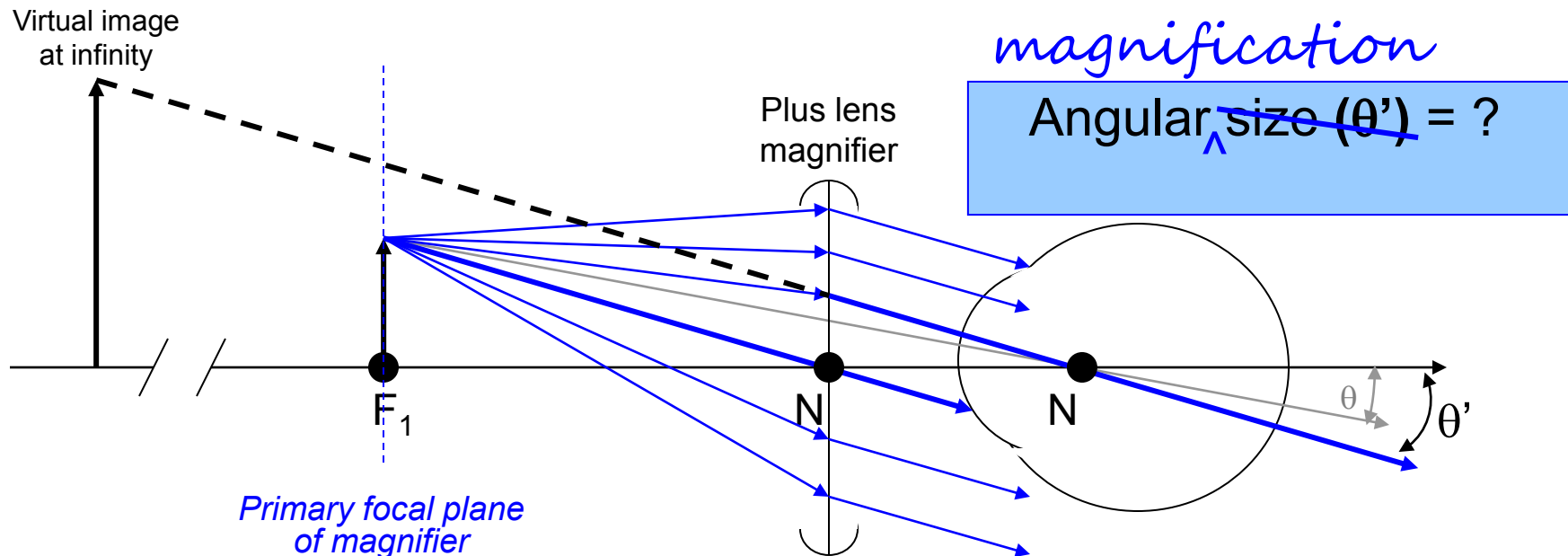- So, angular size is determined by the angular extent of retina an image subtenses (θ)
  - Which, as we have just seen, is a function of object size and lens power

Virtual image at infinity

Plus lens magnifier

Angular size (θ') = *Object size* x *lens power*

F_1

N

N

θ

θ'

Primary focal plane of magnifier

Angular Magnification

- But what about angular *magnification*?

(Remember, size and magnification are not the same thing!)

Virtual image at infinity

Plus lens magnifier

magnification

Angular ~~size~~ (θ') = ?

F_1

N

N

θ

θ'

Primary focal plane of magnifier
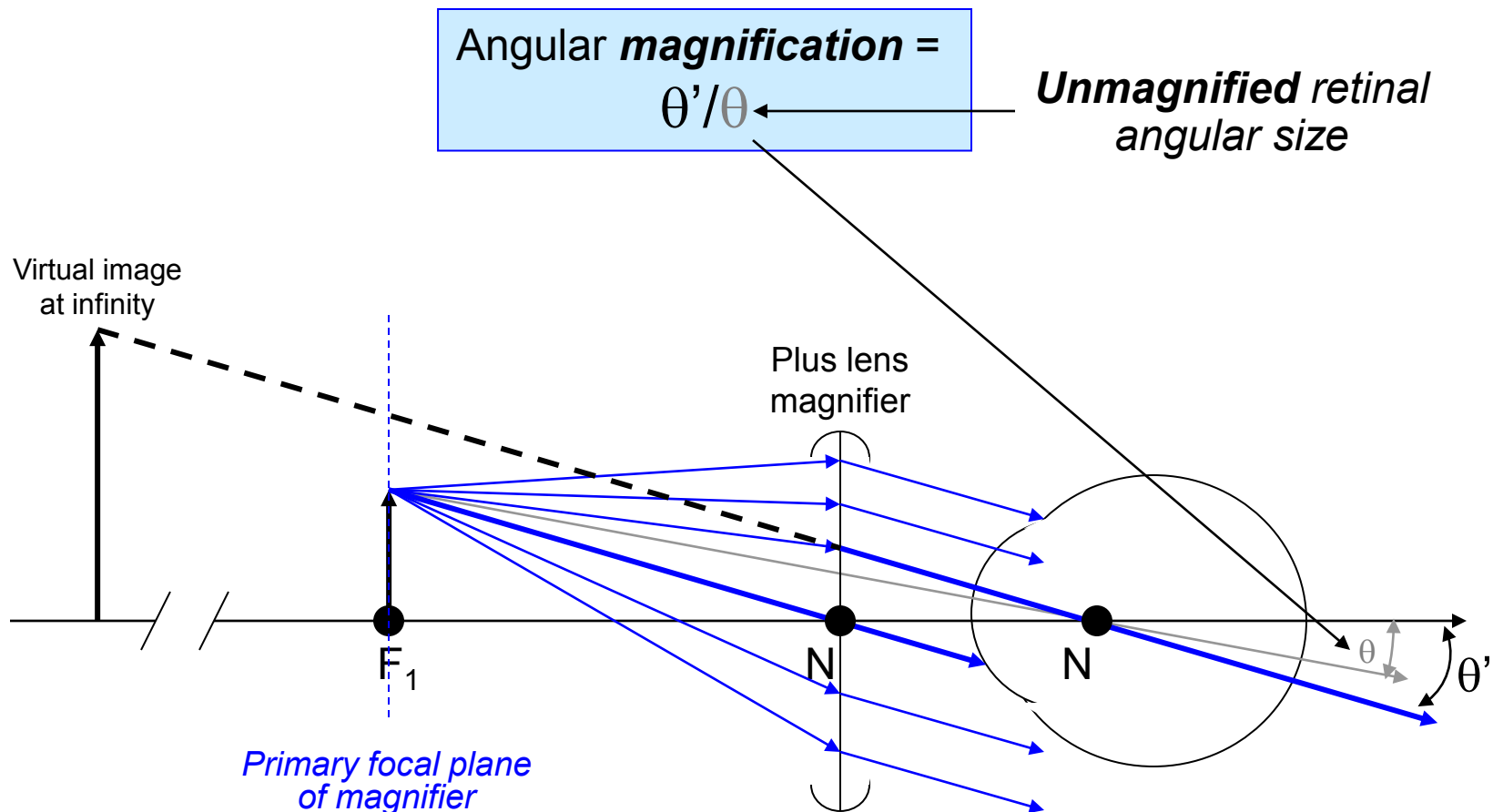
Angular Magnification

- But what about angular *magnification*?
  - 'Magnification' is a relational term, i.e., the retinal image is bigger or smaller relative to *something*

Angular *magnification* = θ'/θ

Unmagnified retinal angular size

Virtual image at infinity

Plus lens magnifier

F_1

N

N

θ

θ'
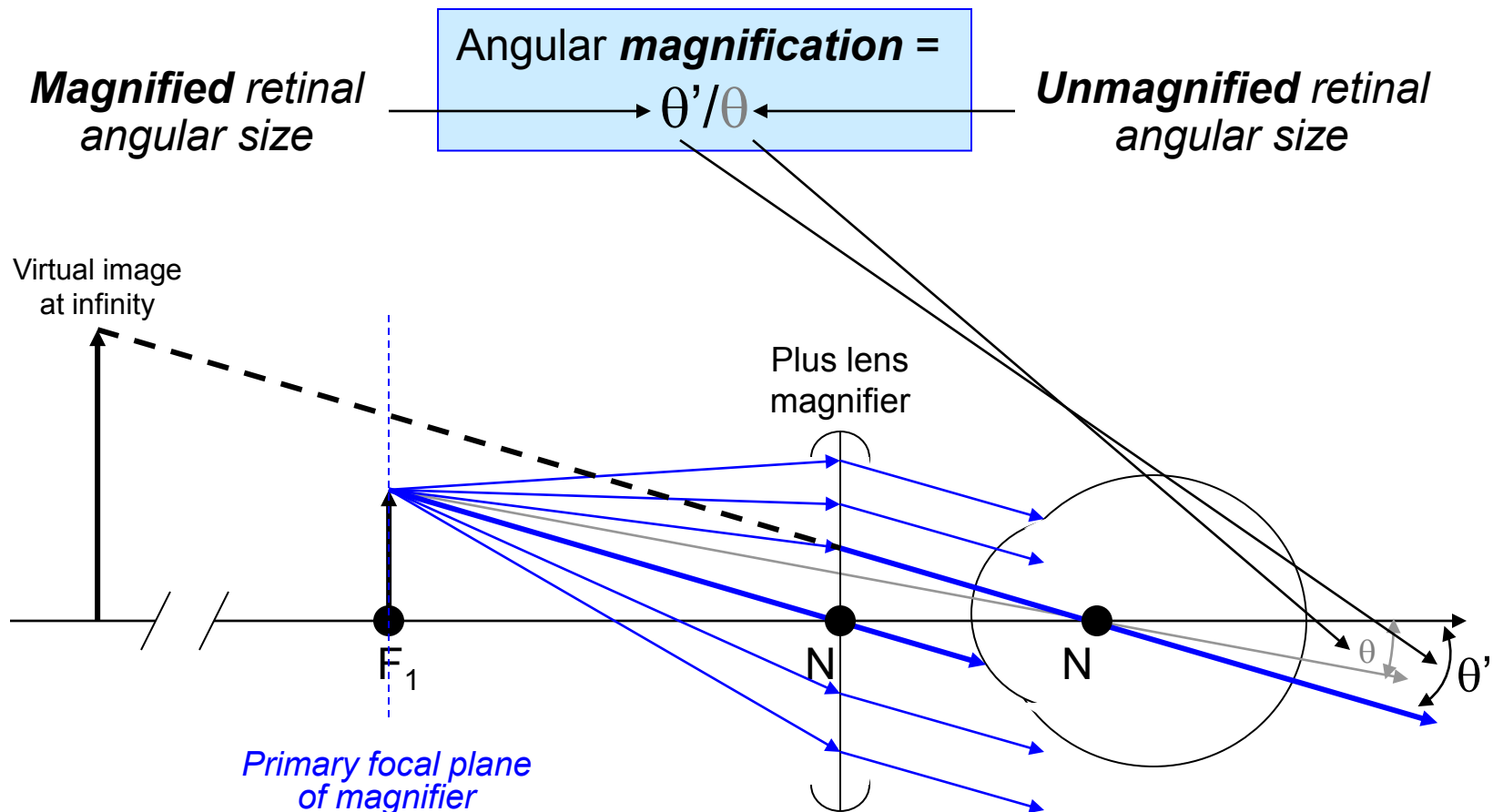
Primary focal plane of magnifier

Angular Magnification

- But what about angular *magnification*?
  - 'Magnification' is a relational term, i.e., the retinal image is bigger or smaller relative to *something*

Magnified retinal angular size

Angular *magnification* = θ'/θ

Unmagnified retinal angular size

Virtual image at infinity

Plus lens magnifier

F_1

N

N

θ

θ'

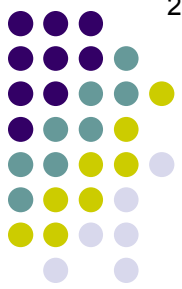*Primary focal plane of magnifier*

Angular Magnification

- But what about angular *magnification*?
  - 'Magnification' is a relational term, i.e., the retinal image is bigger or smaller relative to *something*

Magnified retinal angular size ⟶ | Angular *magnification* = θ'/θ | ⟵ *Unmagnified* retinal angular size

But there's a problem…

Virtual image at infinity

Plus lens magnifier

F_1

Primary focal plane of magnifier
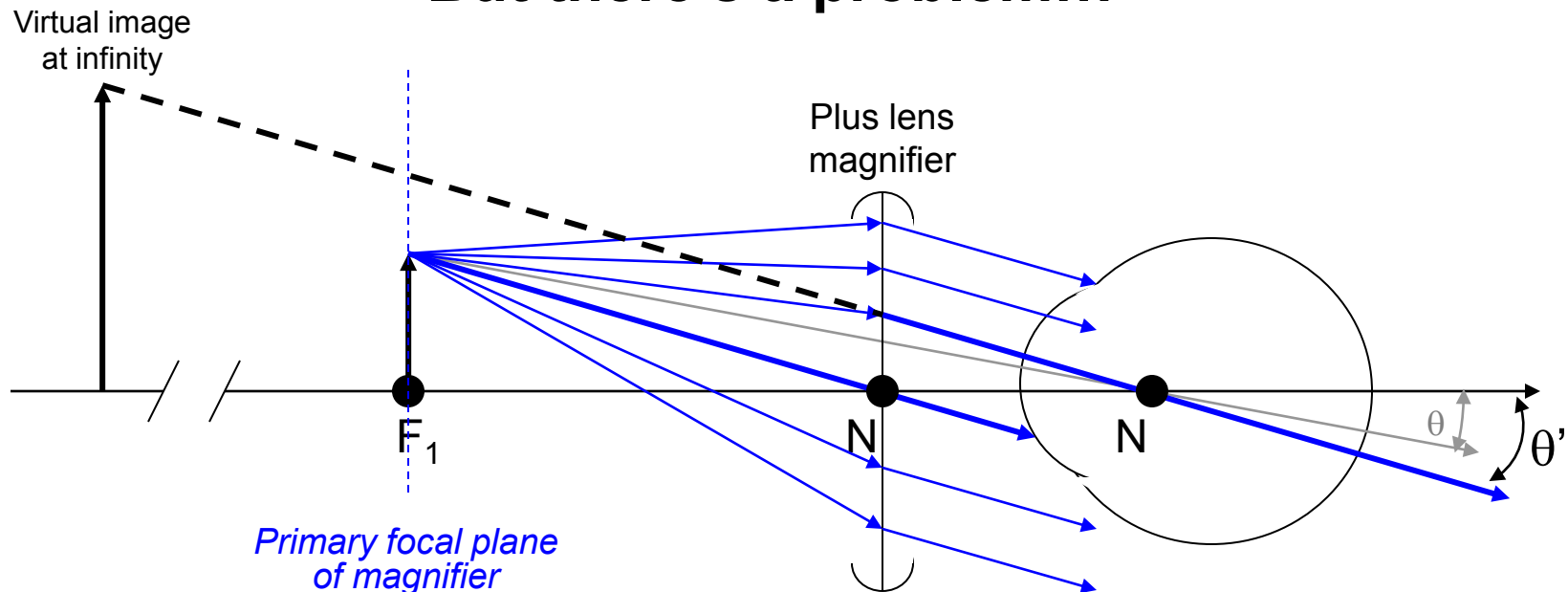
N

N

θ

θ'

Angular Magnification

- But what about angular *magnification*?
  - 'Magnification' is a relational term, i.e., the retinal image is bigger or smaller relative to *something*
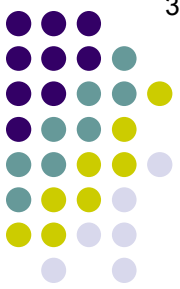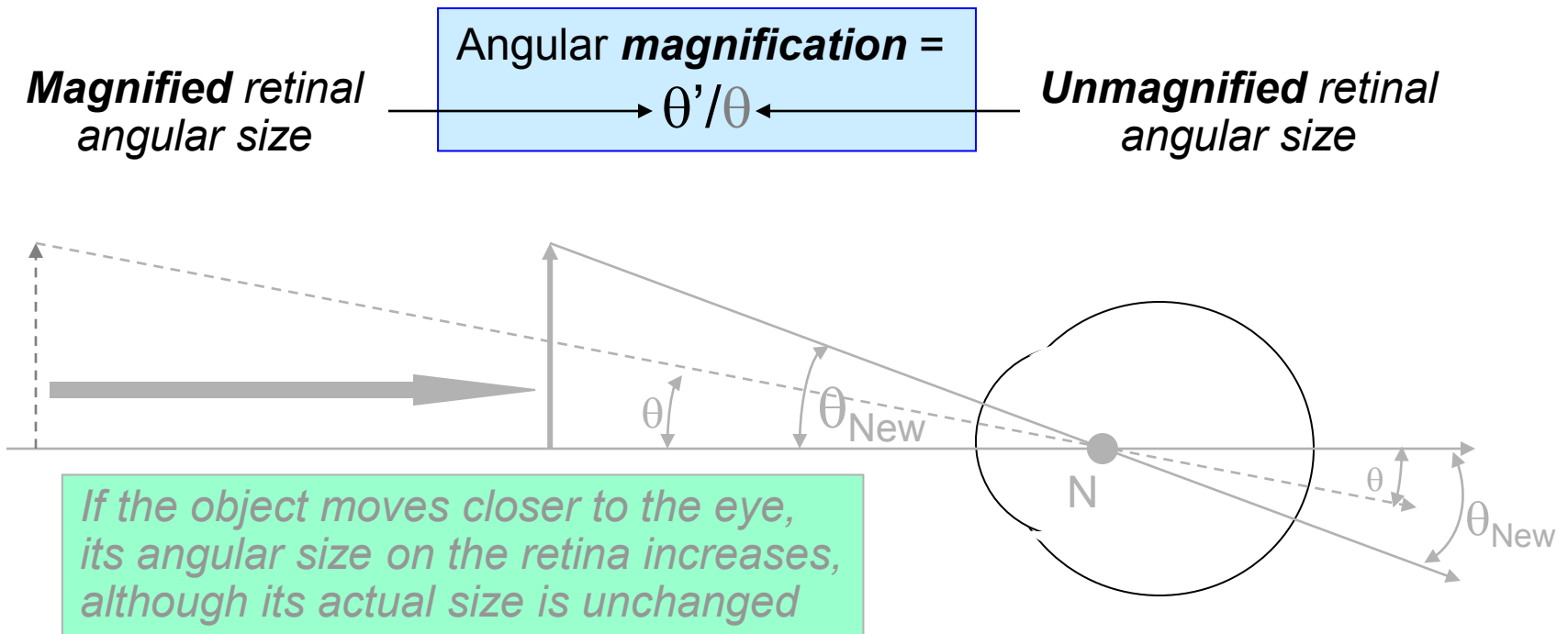
Magnified *retinal angular size* → | Angular **magnification** = θ'/θ | ← **Unmagnified** *retinal angular size*

θ θ_{New} N θ θ_{New}

If the object moves closer to the eye, its angular size on the retina increases, although its actual size is unchanged

Recall this slide, where we saw that the angular size of an image also changes with the *distance* between the object and the retina
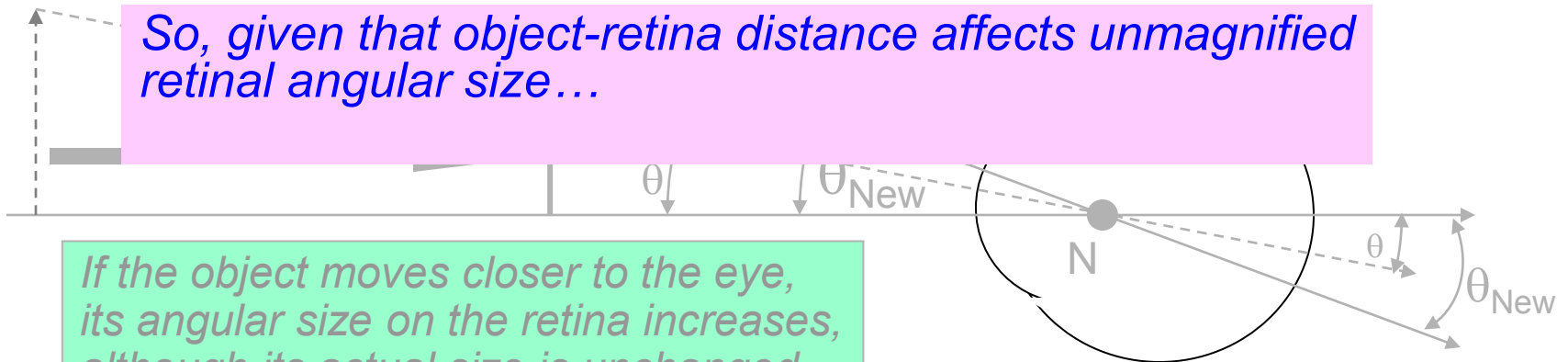
Angular Magnification

- But what about angular *magnification*?
  - 'Magnification' is a relational term, i.e., the retinal image is bigger or smaller relative to *something*

Magnified retinal angular size

Angular *magnification* = θ' / θ

Unmagnified retinal angular size

So, given that object-retina distance affects unmagnified retinal angular size…

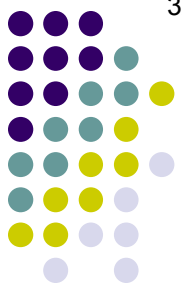θ θ_{New}

N

θ

θ_{New}

If the object moves closer to the eye, its angular size on the retina increases, although its actual size is unchanged

Recall this slide, where we saw that the angular size of an image also changes with the *distance* between the object and the retina
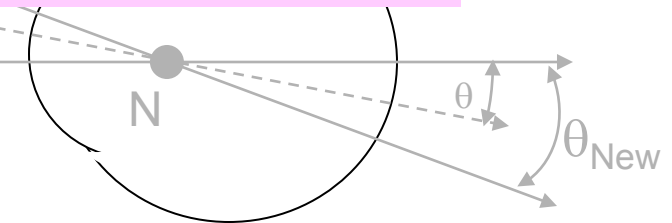
Angular Magnification

- But what about angular *magnification*?
  - 'Magnification' is a relational term, i.e., the retinal image is bigger or smaller relative to *something*

Magnified retinal angular size → Angular *magnification* = θ'/θ ← *Unmagnified retinal angular size*

*So, given that object-retina distance affects unmagnified retinal angular size…**what distance should be used in determining unmagnified retinal angular size?***

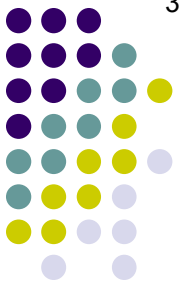θ θ_{New}

N

θ

θ_{New}

If the object moves closer to the eye, its angular size on the retina increases, although its actual size is unchanged

Recall this slide, where we saw that the angular size of an image also changes with the *distance* between the object and the retina
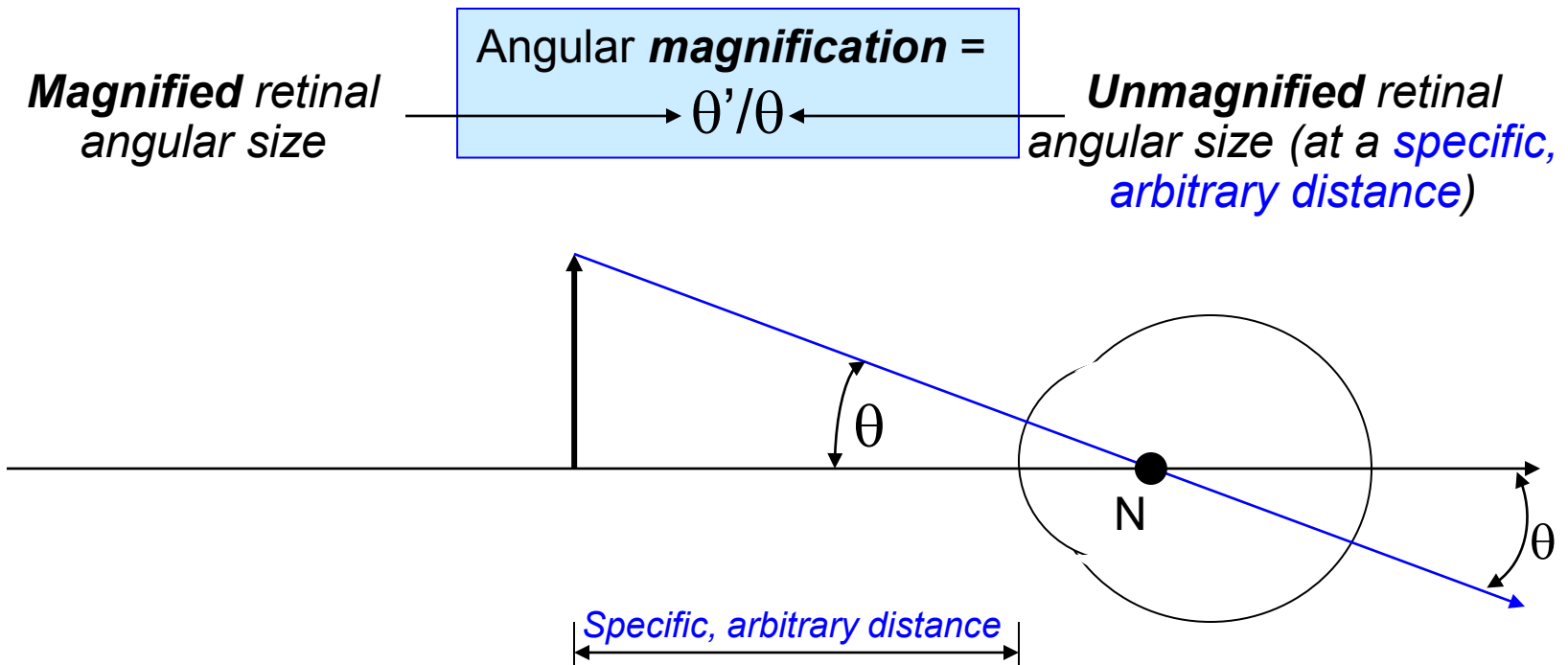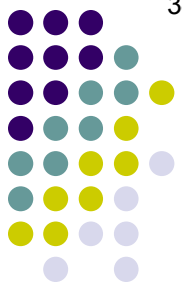
Angular Magnification

- To determine angular magnification, the object must be at a specific, arbitrary distance

Angular Magnification

- To determine angular magnification, the object must be at a specific, arbitrary distance

Angular **magnification** = θ'/θ

Magnified retinal angular size

Unmagnified retinal angular size (at a specific, arbitrary distance)

θ

N

θ

Specific, arbitrary distance

Angular Magnification

- To determine angular magnification, the object must be at a specific, arbitrary distance
  - **A reference distance of *25cm* is the standard**

Magnified retinal angular size ⟶ | Angular *magnification* = θ'/θ | ⟵ *Unmagnified* retinal angular size *at 25cm*

Angular Magnification

Angular **magnification** = θ'/θ

Magnified retinal angular size

Unmagnified retinal angular size at 25cm

Angular Magnification

Angular **magnification** = θ'/θ

Magnified retinal angular size

Unmagnified retinal angular size at 25cm

θ' = Object size x lens power

$\theta = \dfrac{\text{Object size}}{.25m}$
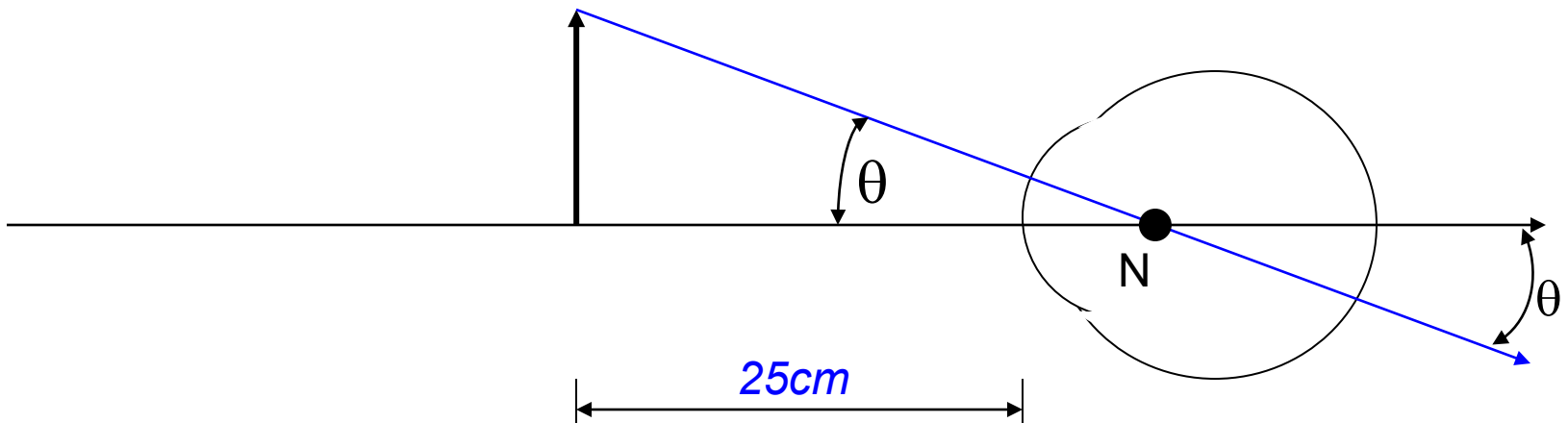
Angular Magnification

Magnified *retinal angular size*

Angular **magnification** = θ'/θ

Unmagnified *retinal angular size at 25cm*

θ' = Object size x lens power

$$\frac{\text{Object size x lens power}}{\dfrac{\text{Object size}}{.25\text{m}}}$$

$\theta = \dfrac{\text{Object size}}{.25\text{m}}$

Angular Magnification

Angular **magnification** =
θ'/θ

Magnified *retinal angular size*

Unmagnified *retinal angular size at 25cm*

θ' = Object size x lens power

~~Object size x lens power~~
~~Object size~~
~~.25m~~

$\theta = \dfrac{\text{Object size}}{.25m}$

Angular Magnification

Angular **magnification** = θ'/θ

Magnified *retinal angular size*

Unmagnified *retinal angular size at 25cm*

$\theta' = $ Object size x lens power

$\dfrac{\text{Object size x lens power}}{\dfrac{\text{Object size}}{.25m}}$

$\theta = \dfrac{\text{Object size}}{.25m}$

$= $ lens power x .25m

Angular Magnification

Angular **magnification** = θ'/θ

Magnified retinal angular size

Unmagnified retinal angular size at 25cm

θ' = Object size x lens power

$$\frac{\text{Object size x lens power}}{\text{Object size}}$$
.25m

$\theta = \dfrac{\text{Object size}}{.25m}$

= lens power x .25m

$$= \frac{\textit{lens power}}{4}$$

Angular Magnification

Angular **magnification** = θ'/θ

Magnified retinal angular size

Unmagnified retinal angular size at 25cm

$\theta' = $ Object size x lens power

$\theta = \dfrac{\text{Object size}}{.25m}$

$$\dfrac{\text{Object size x lens power}}{\dfrac{\text{Object size}}{.25m}}$$

= lens power x .25m

$$= \dfrac{\textbf{lens power}}{\textbf{4}}$$

So, e.g., a 20D lens is a 20/4 = **5x magnifier**