Transverse Magnification

Basic Optics, Chapter 20
Transverse Magnification

- Let’s talk about transverse magnification
  - Also known as *lateral* or *linear* magnification
- Transverse mag concerns the relative height of objects and images in our ray tracings
Transverse Magnification

- Let’s talk about transverse magnification
  - Also known as *lateral* or *linear* magnification
- Transverse mag concerns the relative height of objects and images in our ray tracings
- In principle, with careful tracing, one could simply measure the image and object and determine the ratio directly
  - Fortunately, there are less tedious methods
Transverse magnification is defined as:

\[
\text{Image height} \quad \frac{\text{Object height}}{\text{Image height}}
\]
Transverse magnification is defined as:

\[
\frac{\text{Image height}}{\text{Object height}}
\]

OK, but how do we determine object and image heights when all we have (usually) is info re vergence?
Transverse Magnification

Thin *plus* lens

Here is a ray tracing from a previous chapter.
Here is a ray tracing from a previous chapter. Here it is with only the nodal ray and lens axis ray drawn.
Here is a ray tracing from a previous chapter. Here it is with only the nodal ray and lens axis ray drawn. 

*Think back to high-school geometry—what does the figure look like?*
Transverse Magnification

Thin *plus* lens

![Diagram showing similar triangles and labeled points F1, N, F2 with an angle θ.]

_Similar triangles_
Transverse Magnification

Thin *plus* lens

Diagram showing the relationship between object (O) and image (I) distances, along with the magnification factor (u and v).
Transverse Magnification

Thin *plus* lens

Transverse magnification = $I/O$ (by definition)
Transverse Magnification

Thin \textit{plus} lens

Transverse magnification = \( I/O \) (by definition)

By similar triangles: \( I/O = v/u \)
Thin prs lens

Therefore, transverse magnification is determinable by simply taking a ratio of the image distance to the object distance.

Transverse magnification = \( I/O \) (by definition)
By similar triangles: \( I/O = v/u \)
Transverse Magnification

Thin plus lens

Transverse magnification = \( I/O \) (by definition)

By similar triangles: \( I/O = \frac{v}{u} \)

Therefore, transverse magnification is determinable by simply taking a ratio of the image distance to the object distance

But we can make it more convenient still…
Transverse Magnification

- The Vergence Formula

Recall the Vergence Formula...

\[ u + P = V \]

\[ u = \frac{1}{U} \quad \text{and} \quad v = \frac{1}{V} \]

Vergence of incoming light

Vergence contributed by the lens

Vergence of light leaving lens
Transverse Magnification

- The Vergence Formula

\[ u = \frac{1}{U} \]
\[ v = \frac{1}{V} \]

Recall the **Vergence Formula**...

...and the relationship between vergence (big \( U \), big \( V \)) and distance (little \( u \), little \( v \))

\[ U + P = V \]
Transverse Magnification

\[ U + P = V \]

Thin plus lens

\[ u = 1/U \]
\[ v = 1/V \]

SO, transverse magnification = \( I/O \) (by definition)

AND, by similar triangles, \( I/O = v/u \)

AND, by the Vergence Formula, \( v/u = \frac{1/V}{1/U} = \frac{U}{V} \)
Transverse Magnification

\[ U + P = V \]

Thin *plus* lens

\[ \theta \]

\[ u = \frac{1}{U} \]

\[ v = \frac{1}{V} \]

**SO, transverse magnification** = \( \frac{I}{O} \) (by definition)

**AND**, by similar triangles, \( \frac{I}{O} = \frac{v}{u} \)

**AND**, by the Vergence Formula, \( \frac{v}{u} = \frac{1/V}{1/U} = \frac{U}{V} \)

**THEREFORE**, \( \frac{I}{O} = \frac{U}{V} \)
Transverse Magnification

So, in summary:

\[ U + P = V \]
Transverse Magnification

Transverse magnification is defined as: \( \frac{\text{Image height}}{\text{Object height}} \)

Thin plus lens

Object height

Image height

\[ U + P = V \]
Transverse Magnification

Transverse magnification is defined as:  \[
\frac{\text{Image height}}{\text{Object height}}
\]

Transverse magnification is equal to:

\[U + P = V\]
Transverse Magnification

Transverse magnification is defined as: \( \frac{\text{Image height}}{\text{Object height}} \)

Transverse magnification is equal to:

\[
\frac{v}{u} = \frac{\text{Image distance}}{\text{Object distance}} = \frac{\text{Image height}}{\text{Object height}}
\]

(By similar triangles)

\( u + P = V \)

\( \text{Thin plus lens} \)

\[ \text{Object height} \]

\[ \text{Image height} \]

\[ \text{F}_1 \]

\[ \text{N} \]

\[ \text{F}_2 \]

\[ u \]

\[ v \]
Transverse Magnification

Transverse magnification is defined as: \( \frac{\text{Image height}}{\text{Object height}} \)

Transverse magnification is equal to:

(By the Vergence Law) \( U + P = V \)

(By similar triangles)

Vergence of incoming light (\( U \))
Vergence of light leaving lens (\( V \))

Object distance (\( u \))
Image distance (\( v \))

Thin plus lens

\( U + P = V \)
Transverse Magnification

Transverse magnification is defined as:

\[
\frac{\text{Image height}}{\text{Object height}} = \frac{\text{Image distance (v)}}{\text{Object distance (u)}}
\]

Transverse magnification is equal to:

(By the Vergence Law) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (By similar triangles)

Vergence of incoming light (U)
Vergence of light leaving lens (V)

A few final points about transverse magnification:
Transverse Magnification

Transverse magnification is defined as:

\[
\text{Image height} \quad \frac{\text{Object height}}{\text{Image distance (v)}} \quad \text{Object distance (u)}
\]

Transverse magnification is equal to:

(By the Vergence Law) \quad \frac{\text{Vergence of incoming light (U)}}{\text{Vergence of light leaving lens (V)}} \quad \text{(By similar triangles)} \quad \frac{\text{Image distance (v)}}{\text{Object distance (u)}}

A few final points about transverse magnification:

--The sign of the value indicates the relative orientations of object and image
Transverse Magnification

Transverse magnification is defined as:
\[
\frac{\text{Image height}}{\text{Object height}}
\]

Transverse magnification is equal to:

\[
\frac{V_2 - F_1}{V_1 - F_2} \quad \text{(By the Vergence Law)}
\]

\[
\frac{v}{u} \quad \text{(By similar triangles)}
\]

Vergence of incoming light \((U)\)
Vergence of light leaving lens \((V)\)
Image distance \((v)\)
Object distance \((u)\)

A few final points about transverse magnification:

-- The **sign** of the value indicates the relative orientations of object and image

-- A **positive** value indicates the image has the same orientation as the object
  (i.e., both are either above or below the lens axis)
Transverse Magnification

Transverse magnification is defined as: \[
\frac{\text{Image height}}{\text{Object height}}
\]

Transverse magnification is equal to:

\[
\frac{\text{Vergence of incoming light (U)}}{\text{Vergence of light leaving lens (V)}} = \frac{\text{Image distance (v)}}{\text{Object distance (u)}}
\]

\[\frac{F_2}{F_1} = \frac{N}{N}\]

A few final points about transverse magnification:
--The **sign** of the value indicates the relative orientations of object and image
  --A **positive** value indicates the image has the same orientation as the object
    (i.e., both are either **above** or **below** the lens axis)
  --A **negative** value indicates they are on **opposite** sides of the lens axis
Transverse Magnification

Transverse magnification is defined as:
\[
\frac{\text{Image height}}{\text{Object height}}
\]

Transverse magnification is equal to:

- (By the Vergence Law)
  - Vergence of incoming light (U)
  - Vergence of light leaving lens (V)

- (By similar triangles)
  - Image distance (v)
  - Object distance (u)

A few final points about transverse magnification:
--The size of the value indicates the relative size of object and image
Transverse Magnification

Transverse magnification is defined as:

\[
\frac{\text{Image height}}{\text{Object height}}
\]

Transverse magnification is equal to:

\[
\frac{\text{Image distance}}{\text{Object distance}}
\]

\[
\text{Vergence of incoming light (U)}
\]

\[
\text{Vergence of light leaving lens (V)}
\]

\[
\text{By similar triangles}
\]

\[
\text{By the Vergence Law}
\]

A few final points about transverse magnification:

-- The size of the value indicates the relative size of object and image

-- Transverse mag > 1 → Image is **larger** than the object

---

Diagram showing the relationship between object (O), image (I), and focal points (F1, F2) with transverse magnification of 2.
Transverse Magnification

Transverse magnification is defined as:

\[
\frac{\text{Image height}}{\text{Object height}}
\]

Transverse magnification is equal to:

\[
\frac{\text{Vergence of incoming light (U)}}{\text{Vergence of light leaving lens (V)}} = \frac{\text{Image distance (v)}}{\text{Object distance (u)}}
\]

A few final points about transverse magnification:

-- The **size** of the value indicates the relative size of object and image
-- Transverse mag > 1 \(\rightarrow\) Image is **larger** than the object
-- Transverse mag < 1 \(\rightarrow\) Image is **smaller** than the object

\[
\text{Transverse mag} = 0.5
\]

\[\text{F}_1 \quad \text{N} \quad \text{F}_2\]
Transverse Magnification

Transverse magnification is defined as:
\[
\frac{\text{Image height}}{\text{Object height}}
\]

Transverse magnification is equal to:
\[
\frac{\text{Image distance (v)}}{\text{Object distance (u)}}
\]

\[
\frac{\text{Vergence of incoming light (U)}}{\text{Vergence of light leaving lens (V)}}
\]

A few final points about transverse magnification:
--The **size** of the value indicates the relative size of object and image
  --Transverse mag > 1 \(\rightarrow\) Image is **larger** than the object
  --Transverse mag < 1 \(\rightarrow\) Image is **smaller** than the object
  --Transverse mag = 1 \(\rightarrow\) Image and object are the **same size**

\[
\text{Transverse mag} = -1
\]